HOMOGENEOUS FOCUSING OF ELECTRON BUNCH SEQUENCE BY PLASMA WAKEFIELD

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The mechanism of focusing of relativistic electron bunches by plasma wakefield, in which all bunches of sequence are focused identically and uniformly, has been proposed and investigated. In this scheme of focusing it is necessary that length of each bunch should be equal to a half of wavelength, the charge of first bunch equals half of the charge of the others bunches and the distance between bunches equals 1.5 of wavelength. It is shown that in this case only 1-st bunch is in the finite longitudinal electrical wakefield E_z. Other bunches are in zero longitudinal electrical wakefield E_z= 0. The focusing radial electrical field in regions, occupied by bunches, is constant along each bunch.

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1. INTRODUCTION

The focusing of relativistic electron bunches by wakefield, excited in plasma, is very interesting and important (see, for example, [1-2]). The focusing of bunches by wakefield, excited in plasma by resonant sequence of relativistic electron bunches (repetition frequency of the bunches ω_m coincides with the plasma frequency $\omega_m=\omega_p$), is inhomogeneous. In this paper the specific mechanism of focusing by plasma wakefield, in which all bunches of sequence are focused identically and uniformly, has been proposed and numerically investigated by 2.5D code LCODE [3]. We numerically simulate the self-consistent dynamics of lengthy electron bunches in homogeneous plasma. In simulation we use the hydrodynamic description of plasma. In other words the plasma is considered to be cold electron liquid, and bunches are aggregate of macroparticles.

2. RESULTS OF SIMULATION

We investigate focusing of sequence of electron bunches, length of each bunch equals half of wavelength, because in this case the wakefield has maximal value at increase of bunch length of the same density.

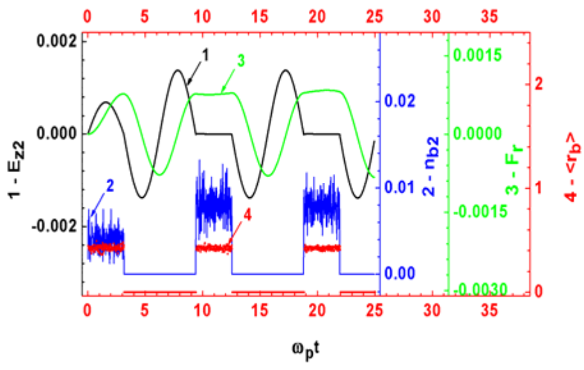

Fig.1. Density of bunches n_b (blue), their averaged radius $<r_b>$ (red), longitudinal wakefield E_z (black) and radial wake force F_r (green)

We use the charge of first bunch, equal in two times smaller than charge of each next bunch

$$Q_1=Q/2,\ Q_i=Q,\ i=2, 3, 4 ...,$$

and distance between bunches equals 1.5λ

$$\xi_{i+1}-\xi_i=1.5\lambda,$$

λ is the wakefield wavelength. Then for three bunches the distribution of bunch density n_b, their averaged radius $<r_b>$, excited longitudinal wakefield E_z and radial wake force F_r are of the form, shown in Fig.1. One can see that all bunches (which are in $E_z = 0$) with the exception of first one (which are in $E_z \neq 0$) do not change by energy with longitudinal wakefield. Then wakefield after i-th bunch is the same as before it (with the exception of first bunch). But the bunches are focused because radial wake force is not equal zero $F_r \neq 0$. This wakefield lens has following qualities:

1) radial wake force F_r does not approximately depend on coordinate in regions, occupied by bunches (with the exception of first bunch), $F_r \approx$ const, i.e. lengthy bunches are focused identically;
2) only first bunch is slowed down;
3) identical focusing force effects on all bunches (with the exception of first bunch);
4) longitudinal wakefield equal zero $E_z = 0$ in regions, occupied by bunches (with the exception of first bunch).

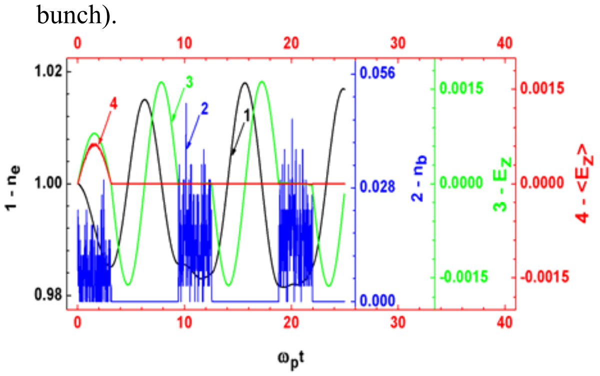

Fig.2. Plasma electron density n_e (black) in wakefield, longitudinal wakefield E_z (green), density of bunches n_b (blue) and $<E_z>=\int dr\, r\, E_z n_b / \int dr\, r n_b$ (red) coupling rate of bunch with wakefield E_z

Such ideal focusing is realized due to formation of flat holes of plasma electron density n_e in regions, occupied by bunches (Fig.2), which neutralize charges of bunches and focuse them. In region, occupied by first bunch (see Fig.2), the formed hole of plasma electron density n_e is not flat. One can see from Fig.2 that wave of plasma electron density n_e is nonsinusoidal: wide holes of n_e (with the exception of region, occupied by first bunch) are interchanged by narrow humps of n_e. The bunches are located in the regions of the holes of n_e.

In Fig.1 one can see that the usual for linear case connection of radial and longitudinal wakefields $E_r \sim \partial_r E_z$ is not true.

In Fig.3 one can see that on wavelength (regions a and b) the positive and negative perturbations of E_z compensate each other, hence on the last part (region c) of wave period ($\lambda/2$), where bunches are localized, longitudinal wakefield E_z can become zero (it is observed). Radial wake force F_r does not equal zero $F_r \neq 0$ in regions, occupied by bunches.

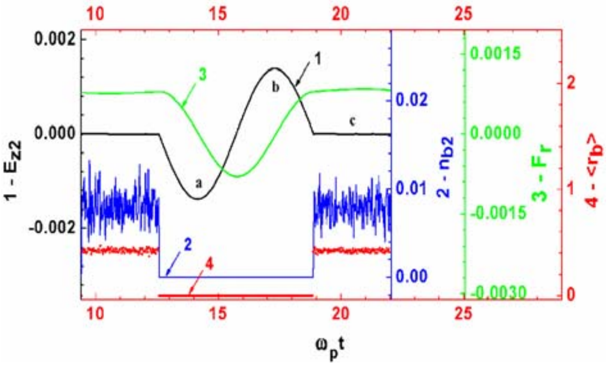

Fig.3. Longitudinal wakefield E_z (black) and radial wake force F_r (green) on one wave period

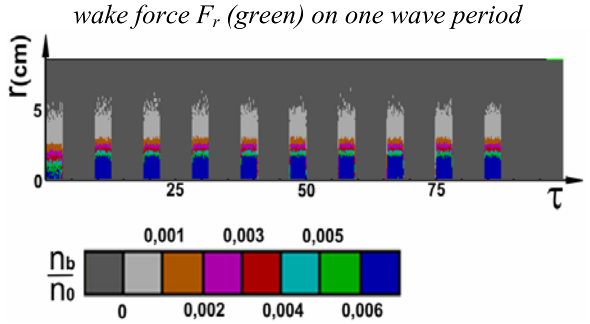

Fig.4. The density of ten bunches

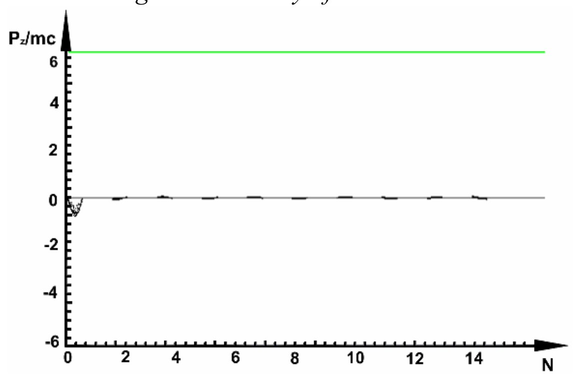

Fig.5. Change of longitudinal momentum of bunches P_z at wakefield excitation. N is the number of wavelengths

These results are right and for larger number of bunches. Let us consider ten bunches (Fig.4). Again the charge of first bunch is two times smaller than charge of each next bunch $Q_1=Q/2$, $Q_i=Q$, i=2, 3, 4 ... and distance between bunches equals $\xi_{i+1}-\xi_i=1.5\lambda$. Then the spatial distributions of plasma electron density n_e, density of bunches n_b, their averaged radius $\langle r_b\rangle$, excited longitudinal wakefield E_z, radial wake force F_r, coupling rate of bunch with longitudinal wakefield $\langle E_z\rangle$ and longitudinal momentum of bunches P_z have views, shown in Figs.4-8. From Fig.6 one can see that $\langle E_z\rangle$ of only 1-st bunch does not equal zero and therefore only 1-st bunch is slowed down (see Fig.5).

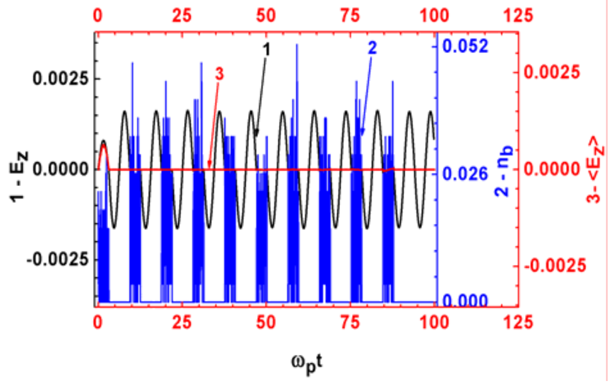

Fig.6. The density of bunches n_b (blue), longitudinal wakefield E_z (black) and $\langle \boldsymbol{E_z}\rangle$ (red) coupling rate of bunch with wakefield E_z

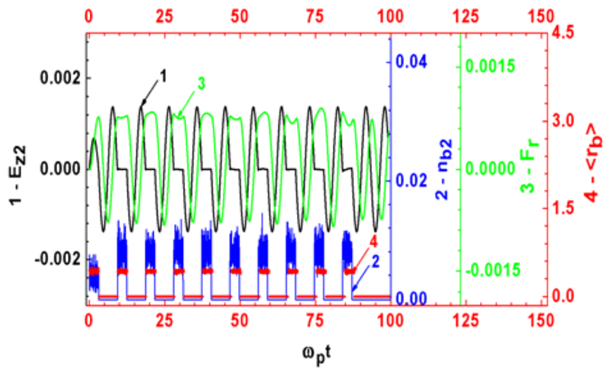

Fig.7. The density of bunches n_b (dark blue), their averaged radius $\langle r_b\rangle$ (red), longitudinal wakefield E_z (black), radial wake force F_r (green)

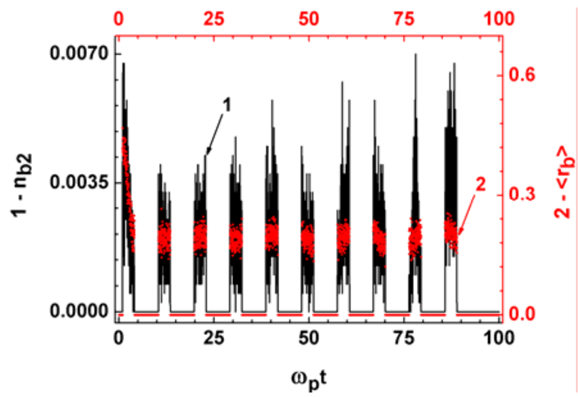

Fig.8. The density of bunches n_b (black) and their averaged radius $\langle r_b\rangle$ (red) after focusing

Again one can (see Fig.7) that only 1-st bunch is in finite $E_z \neq 0$. Other bunches are in zero longitudinal electrical wakefield $E_z = 0$. Radial wake force F_r in regions, occupied by bunches, is finite (see Fig.7). In Fig.8 one can see that radius of bunches (and their density n_{b2} on r, equal initial radius of bunches) decreases due to focusing.

We considered bunches with close radius and length. If the bunches are represented «needles», i.e. if their lengths are larger than radius, then plateau-electron-holes (Fig.10), F_r longitudinal distribution in regions, occupied by bunches (Fig.9), and focusing (Fig.11) are more ideal.

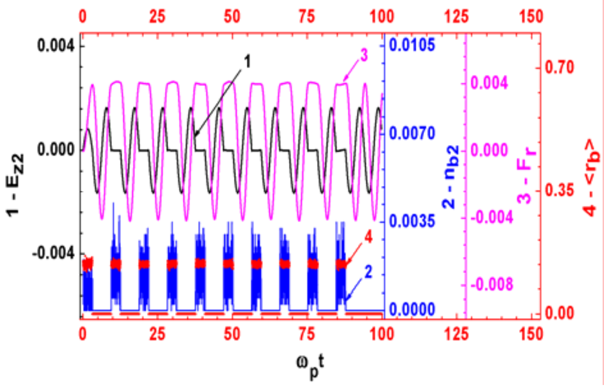

Fig.9. The density of needle bunches n_b (blue), their averaged radius $<r_b>$ (red), longitudinal wakefield E_z (black) and radial wake force F_r (pink)

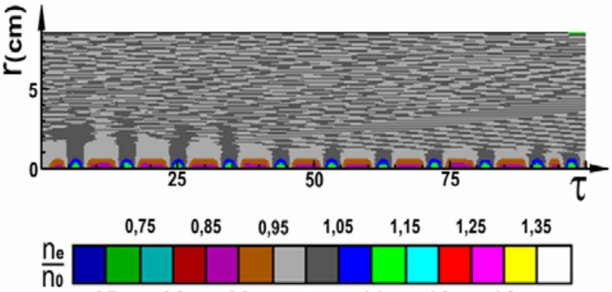

Fig.10. Plasma electron density n_e in wakefield, excited by needle bunches

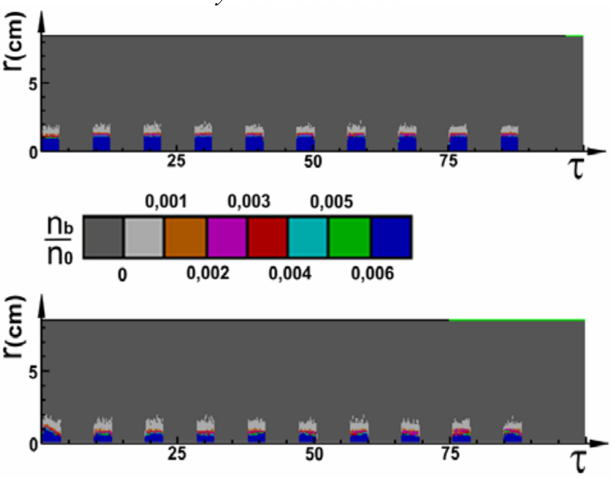

Fig.11. The density n_b of ten bunches before and after focusing

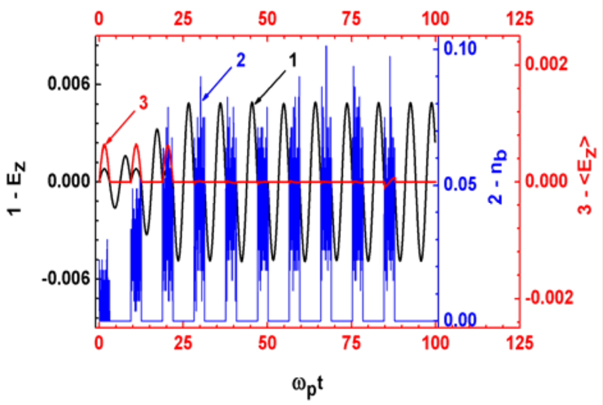

Fig.12. Longitudinal wakefield E_z (black), density of bunches n_b (blue) and $<\boldsymbol{E_z}>$ (red) coupling rate of bunch with wakefield E_z

For radial wake force F_r enhancement let us consider following shaped sequence of bunches. The charges of first n = 3 shaped bunches increase along sequence according to: 2k-1, k≤n (Fig.12,14). The charges of next bunches equal 2n, k>n. Then the distribution of plasma electron density n_e, density of bunches n_b, their averaged radius $<r_b>$, excited longitudinal wakefield E_z, radial wake force F_r, coupling rate of bunch with longitudinal wakefield $<E_z>$ and longitudinal momentum of bunches P_z have views, shown in Figs.12-14.

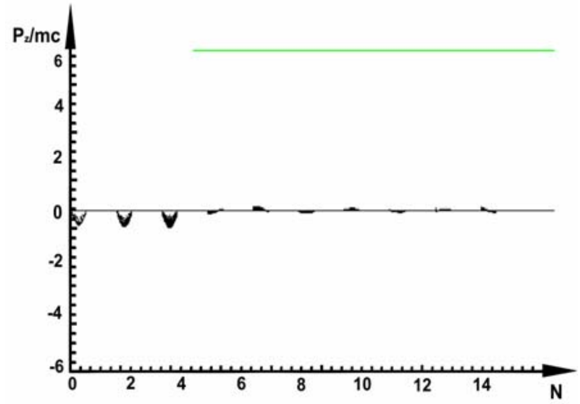

Fig.13. Change of longitudinal momentum of bunches P_z at wakefield excitation. N is the number of wavelengths

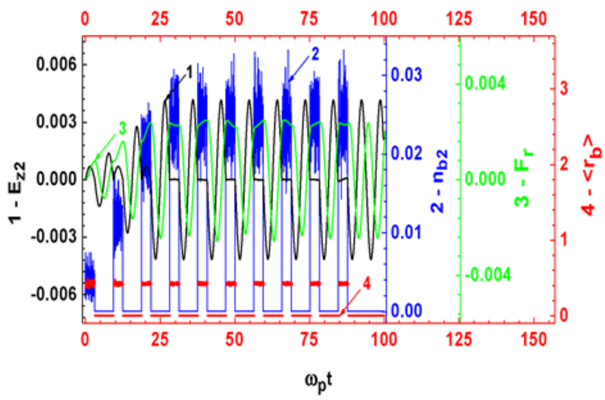

Fig.14. The density n_b (blue) of shaped bunches, their averaged radius $<r_b>$ (red), longitudinal wakefield E_z (black) and radial wake force F_r (green)

From Fig.12 one can see that the coupling rates <E> of only first three bunches n = 3 with longitudinal wakefield E_z do not equal zero and therefore only first three bunches are slowed down (see Fig.13).

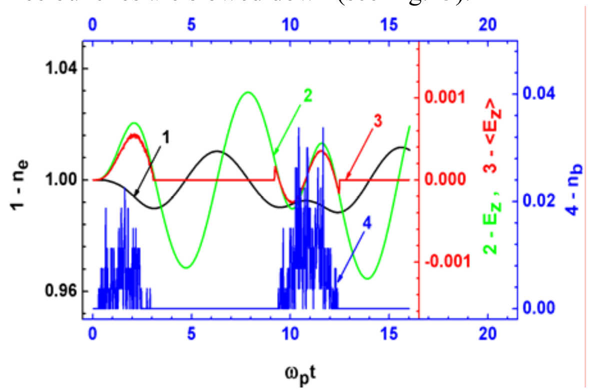

Fig.15. Plasma electron density n_e (black), longitudinal wakefield E_z (green), density n_b (blue) of two bunches and $<\boldsymbol{E_z}>$ (red) coupling rate of bunch with wakefield E_z

From Fig.14 one can see that radial wake force F_r does not depend approximately on coordinate in regions, occupied by bunches k>n=3, $F_r \approx$ const, i.e. the lengthy bunches are focused approximately identically.

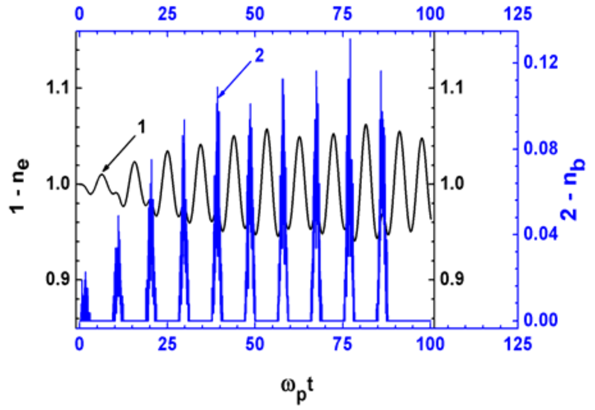

Fig.16. The distribution of plasma electron density n_e (grey) in wakefield

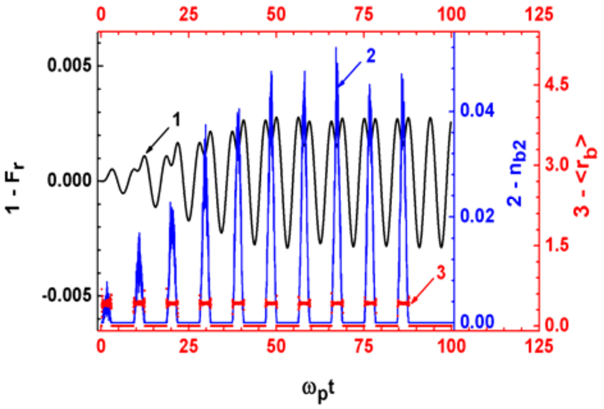

Fig.17. The density of bunches n_b (blue), their averaged radius $<r_b>$ (red) and radial wake force F_r (black)

It is for ideal case, when bunches are represented by cylinders of constant density in longitudinal direction, i.e. n_b=const. The bunch electrons are distributed in radial direction according to Gauss distribution. However the bunch density n_b distribution is inhomogeneous in longitudinal direction. Because Gauss distribution and cosine distribution are close, we consider the cosine electron density distribution along bunch (Figs.15-17). From Fig.15 one can see that in this case of inhomogeneous longitudinal distribution of electron bunch density the 1-st front of 2-nd bunch is accelerated and back front is decelerated. I.e. the 2-nd bunch is in $E_z \neq 0$.

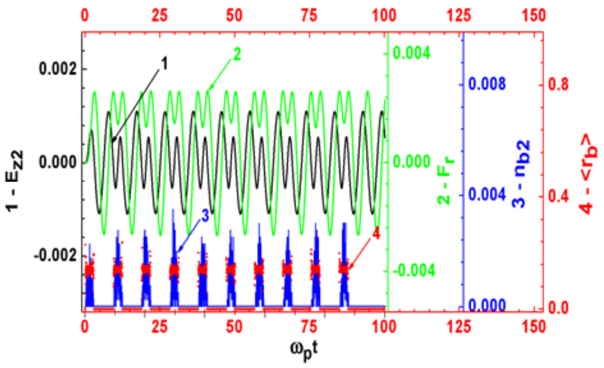

Fig.18. The density n_b (blue) of inhomogeneous needle bunches, their averaged radius $<r_b>$ (red), longitudinal wakefield E_z (black) and radial wake force F_r (green)

The distribution of radial wake force F_r in the case of inhomogeneous n_b distribution along needle bunch is shown in Fig.18. From Fig.17 and Fig.18 one can see that the middle of bunch is focused more slower than fronts.

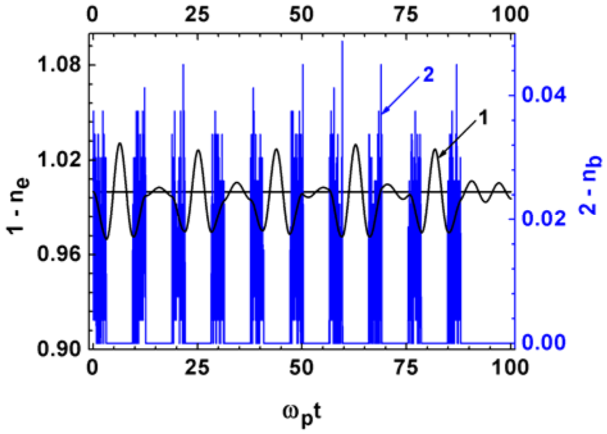

Fig.19. Plasma electron density n_e (black) in wakefield and density of bunches n_b (blue)

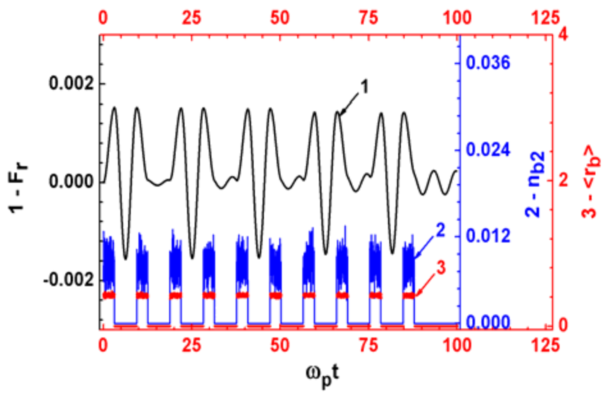

Fig.20. The density of bunches n_b (dark blue), their averaged radius $<r_b>$ (red) and radial wake force F_r (black)

If all bunches are identical and they are distributed through 1.5λ, then all bunches are focused identically, but inhomogeneously along each bunch (Figs.19, 20).

One can show that the field distribution for positron bunches are identical to electron bunches and they are focused identically to electron bunches.

CONCLUSIONS

It has been shown that all bunches of sequence can be focused identically and uniformly. For this it is necessary that bunches have lengths, equal λ/2, the charge of 1-st bunch equals half of the charges of the other bunches, the distance between them equals 1,5 λ. It is shown that only 1-st bunch is in finite $E_z \neq 0$. Other bunches are in zero longitudinal electrical wakefield $E_z = 0$. Hence the 1-st bunch interchange by energy with wakefield. The subsequent bunches do not interchange by energy with wakefield and the amplitude of wakefield does not change along sequence. Radial wake force F_r in regions, occupied by bunches, is approximately constant along bunches.

In the case of inhomogeneous longitudinal distribution of electron bunch density the middle of bunches are focused more slower than fronts.

If all bunches are identical and they are distributed through 1,5λ, then all bunches are focused identically, but inhomogeneously along each bunch.

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ОДНОРОДНАЯ ФОКУСИРОВКА ПОСЛЕДОВАТЕЛЬНОСТИ ЭЛЕКТРОННЫХ СГУСТКОВ КИЛЬВАТЕРНЫМ ПОЛЕМ В ПЛАЗМЕ

К.В. Лотов, В.И. Маслов, И.Н. Онищенко, Е.Н. Свистун

Предложен и исследован механизм фокусировки релятивистских электронных сгустков плазменными кильватерными полями, в которых все сгустки последовательности фокусируются одинаково и однородно. В этой схеме фокусировки необходимо, чтобы длина каждого сгустка была равна половине длины кильватерной волны, заряд первого сгустка был в 2 раза меньше заряда каждого из остальных сгустков, а расстояние между ними составляло 1,5 длины волны. Показано, что в этом случае только на 1-й сгусток действует продольное поле E_z. Остальные сгустки находятся в нулевом продольном кильватерном поле $E_z = 0$. Фокусирующее радиальное поле в областях, занятых сгустками, постоянно по длине сгустков.

ОДНОРІДНЕ ФОКУСУВАННЯ ПОСЛІДОВНОСТІ ЕЛЕКТРОННИХ ЗГУСТКІВ КІЛЬВАТЕРНИМ ПОЛЕМ У ПЛАЗМІ

К.В. Лотов, В.І. Маслов, І.М. Оніщенко, О.М. Свистун

Запропонований і досліджений механізм фокусування релятивістських електронних згустків плазмовими кільватерними полями, у яких усі згустки послідовності фокусуються однаково та однорідно. В цій схемі фокусування необхідно, щоб довжина кожного згустка дорівнювала половині довжини кільватерної хвилі, заряд першого згустку був у 2 рази менше заряда кожного із решти згустків, а відстань між ними була рівною 1,5 довжини хвилі. Показано, що в цьому випадку лише на 1-й згусток діє повздовжнє електричне поле E_z. Останні згустки знаходяться у нульовому повздовжньому кільватерному полі $E_z = 0$. Фокусуюче радіальне електричне поле в зонах, що заняті згустками, постійне по довжині згустків.