Section F. Nonlinear Dynamics and Chaos

EXCITATION AND PROPERTIES OF LARGE AMPLITUDE SOLITON NEAR FOIL AT LASER PULSE INTERACTION WITH IT

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The properties and excitation of solitary hump of electric potential of large amplitude, observed in experiment and propagating with thermal velocity of plasma electrons near foil at laser pulse interaction with it, have been described. This soliton has been observed at interaction of laser pulse of duration 1 nsec of power 10^{14} W/cm^2 with foil of thickness 26 μm. The excitation of soliton by nonstationary electrical field has been described for the first time. The properties of soliton has been described in strongly nonlinear case. The dependences of width and velocity of soliton on its amplitude at large values of amplitude have been derived. Both in experiments and numerical simulation it was shown that the width of soliton increases with amplitude growth at large values of amplitude.

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1. INTRODUCTION

The interaction of an intense laser pulse with a foil with the purpose of ion acceleration is an issue currently under investigation worldwide [1-2]. These ion beams can be used for a range of applications, for example in fast ignition and for neutral beam injection in nuclear fusion devices. Laser acceleration of ions was observed from 1960s [3], but the interest on this topic has increased dramatically in recent years due to experimental access to relativistic interaction regimes and increased efficiency in energy coupling from the laser pulse to energetic electrons. Earlier the formation of virtual cathode, electrical double layer, semi-vortex and vortex near foil at powerful laser pulse interaction with it has been investigated. Recently the soliton has been observed in [4] at interaction of laser pulse of duration 1 nsec of power 10^{14} W/cm^2 with foil of thickness 26 μm. In this paper the properties and excitation of this solitary hump of electric potential of large amplitude, observed in experiments [4] and propagating with thermal velocity of plasma electrons near foil at laser pulse interaction with it, have been described analytically. The excitation of soliton by nonstationary electrical field has been described analytically for the first time. The properties of soliton have been described in strongly nonlinear case. It has been shown that with amplitude growth the width of soliton increases at large values of amplitude as in experiments and numerical simulation.

2. KINETIC DESCRIPTION OF SMALL AMPLITUDE SOLITON ON TIME, LESS THAN TRAPPED ONE

We consider the solitary perturbation of electrical potential. We describe its in 1D approximation. We suppose that it moves with velocity V_s, close to thermal velocity V_{th} of plasma electrons, along z. We describe the solitary perturbation of electrical potential φ of amplitude, φ_0. Because at first we look for the solution of stationary soliton, moving with velocity V_s, we use the dependence of the electron distribution function f_e on coordinate z and time t in kind $\xi = z - V_s t$.

We look for the solution of kinetic equation for electron distribution function f_e of kind:

$$f_e = f_0 + \delta f, f_0 = \frac{n_0}{V_{th}(2\pi)^{1/2}} \exp\left(-\frac{V^2}{2V_{th}^2}\right),$$
$$\delta f = \delta f(\xi, \tau), \qquad \xi = z - tV_s,$$
$$\delta f(\xi, \tau) = \delta f(\xi) + \delta f_\tau, \delta f(\xi) = \delta f_L + \delta f_{NL}. \quad (1)$$

Here f_0 is the unperturbed distribution function, δf is the perturbation of plasma electron distribution function, ∂_τ is the time derivative, $\delta f(\xi)$ is the qausistationary solution, δf_L, δf_{NL} are the linear and nonlinear parts, δf_τ is the part, determined by time dependence of soliton at its excitation, δf_τ is proportional to $\partial_\tau \varphi$:

$$\delta f_L = -\frac{e}{m_e}\varphi(V - V_s)^{-1}\partial_v f_0,$$
$$\delta f_{NL} = \left(\frac{e}{m_e}\right)^2 \frac{\varphi^2}{2}(V - V_s)^{-1}\partial_v f_0. \quad (2)$$

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Integrating the latter expression on velocity and introducing them in Poisson equation, one can derive:

$$\partial_\eta^2\varphi = \varphi R(v_s) + \frac{\varphi^2}{2}\left[\left(\frac{3}{2} - v_s^2\right)R(v_s) - \frac{1}{2}\right], \quad (3)$$

$$\varphi = \frac{e\varphi}{T_e},\ v_s = \frac{V_s}{V_{th}2^{1/2}},\ y = \frac{V}{V_{th}2^{1/2}},\ \eta = \frac{\xi\omega_{pe}}{V_{th}},$$

$$R(v_s) = 1 + v_s\pi^{-1/2}\int\limits_{-\infty}^{\infty} dy(y - v_s)^{-1}\exp(-y^2).$$

Here ω_{pe} is the Langmuir frequency of plasma electrons, T_e is the temperature of plasma electrons. From this equation we derive equation:

$$(\partial_\eta\varphi)^2 = \varphi^2 R(v_s) + \frac{\varphi^3}{3}\left[\left(\frac{3}{2} - v_s^2\right)R(v_s\right) - \frac{1}{2}\right]. \quad (4)$$

From the latter equation and natural condition, $\partial_\eta\varphi \mid_{\varphi=\varphi_0} = 0$, we obtain expression for v_s (similar to [4]):

$$v_s \simeq 0.924\left(1 - \frac{\varphi_0}{6}\right), \quad V_s \simeq 1.3V_{th}\left(1 - \frac{e\varphi_0}{6T_e}\right). \quad (5)$$

V_s equals approximately V_{th}. From Poisson equation we also derive similar to [4] the expression for the soliton width:

$$\Delta\eta \simeq \frac{\varphi_0}{[(\partial_\eta\varphi)^2 \mid_{\varphi=\varphi_0/2}]^{1/2}} \simeq \frac{7}{\varphi_0^{1/2}}. \quad (6)$$

3. SOLITON EXCITATION BY QAUSISTATIONARY ELECTRICAL FIELD

We consider the soliton excitation by nonstationary electrical field $E_0(t)$. From Poisson equation one can obtain the part δf_τ, determined by time dependence of soliton at its excitation:

$$\begin{aligned}\partial_\eta\delta f_\tau = &-\left(\frac{e}{m_e}\right)^2\varphi E_0(t)(V - V_s)^{-1}\\ &\times \partial_v(V - V_s)^{-1}\partial_v f_0\\ &+ \left(\frac{e}{m_e}\right)E_0(t)(V - V_s)^{-1}\partial_v f_0\\ &+ \left(\frac{e}{m_e}\right)\partial_\tau\varphi(V - V_s)^{-2}\partial_v f_0. \quad (7)\end{aligned}$$

Here e and m_e are the electron charge and mass. Integrating the latter expression on velocity, we derive add to the perturbation of plasma electron density:

$$\begin{aligned}\partial_\eta\delta n_\tau = &-\left[\frac{n_0}{V_{th}2^{1/2}}\right]\partial_\tau\varphi\mu - \left(\frac{e}{T_e}\right)n_0\varphi E_0(t)\nu\\ &-\left(\frac{e}{T_e}\right)n_0E_0(t)R(v_s),\\ \nu = &-\frac{1}{2} + \left(\frac{3}{2} - v_s^2\right)R(v_s),\\ \mu = &-\frac{1}{v_s} + R(v_s)\left(\frac{1}{v_s} - 2v_s\right). \quad (8)\end{aligned}$$

Because from Poisson equation it follows:

$$\delta n_\tau = 0, \quad (9)$$

we derive growth rate γ_{NL} of soliton amplitude φ_0:

$$\gamma_{NL} = \varphi_0^{-1}\partial_\tau\varphi_0 \simeq -0.65\left(\frac{e}{V_{th}m_e}\right)E_0(t), E_0 \leq 0. \quad (10)$$

4. SOLITON PROPERTIES IN ADIABATICAL APPROXIMATION

We consider dependences of soliton properties on amplitude for times, larger than electron interaction time with it, i.e. larger than electron transit time through soliton. In this approximation the qausistationary distribution function of electrons f_e on velocity V has the following form:

$$f_e = f_{0e}\left[\left(u^2 - \frac{2e\varphi}{m_e}\right)^{1/2} + V_s\right], \quad u \geq A(\varphi)\mathrm{sign}(z),$$

$$f_e = f_{0e}\left[\left(u^2 - \frac{2e\varphi}{m_e}\right)^{1/2} - V_s\right], \quad u \leq A(\varphi)\mathrm{sign}(z),$$

$$f_e = f_{tr} = \frac{n_{tr}}{V_{tr}(2\pi)^{1/2}}\exp\frac{(e\varphi - mu^2/2)}{T_{tr}}. \quad (11)$$

It means that trapped electrons are distributed near separatrix. Here $A(\varphi) = (2e\varphi/m_e)^{1/2}$ is the width of resonance on plasma electron velocity, n_{tr} and T_{tr} are the trapped electron density and temperature, V_{tr} is the width of the trapped electron distribution function on velocity. On large time due to relaxation the condition of distribution function continuity can be correct on separatrix, where energy equals zero $\epsilon = 0$:

$$f_{0e}\mid_{\epsilon=0} = f_{tr}\mid_{\epsilon=0}. \quad (12)$$

From here we derive the connection of parameters:

$$f_{0e}(V_s) = \frac{n_{tr}}{V_{tr}(2\pi)^{1/2}}, \quad \frac{n_0}{V_{th}}\exp\left(-\frac{V_s^2}{2V_{th}^2}\right) = \frac{n_{tr}}{V_{tr}}.$$

Integrating f_e on velocity, one can derive the expression for the electron density n_e:

$$\begin{aligned}n_e = &\,2\exp\left(\frac{e\varphi}{T_{tr}}\right)\int\limits_0^A du f_{tr}(u)\\ &+ \left(\frac{1}{\pi^{1/2}}\right)\int\limits_{-\infty}^{\infty} dt\frac{\exp(-t^2)}{[1 + \varphi/(t - v_s)^2]^{1/2}},\\ v_s = &\,\frac{V_s}{V_{th}2^{1/2}}, \varphi = \frac{e\varphi}{T_e}. \quad (13)\end{aligned}$$

Not taking into account trapped electrons in approximation of small amplitude we obtain:

$$\begin{aligned}n_e \simeq &\, n_0[1 + \varphi R(v_s) + (\varphi/2)^2\\ &\times [2 - 2v_s^2 + (3 - 2v_s^2)(R(v_s) - 1)]], \quad (14)\end{aligned}$$

$$R(v_s) = 1 + \frac{v_s}{\pi^{1/2}} \int_{-\infty}^{\infty} dt \frac{\exp(-t^2)}{(t - v_s)}.$$

This has been derived earlier from kinetic equation in "quick" approximation, i.e. on times, shorter than trapped one. In the case of small amplitude the adiabatic approximation without taking into account trapped electrons and "quick" approximation lead to the same result.

5. THE DEPENDENCE OF SOLITON VELOCITY ON AMPLITUDE AT LARGE ITS VALUES

Let us consider strongly nonlinear case, $\varphi_0 \gg 1$, without taking into account trapped electrons. From Poisson equation we derive:

$$\frac{(\partial_\eta \varphi)^2}{2} = -\varphi + \frac{2}{\pi^{1/2}} \int_{-\infty}^{\infty} dt (t - v_s)^2 \exp(-t^2) \times \left[\left[1 + \frac{\varphi}{(t - v_s)^2} \right]^{1/2} - 1 \right]. \quad (15)$$

From condition $\partial_\eta \varphi \mid_{\varphi = \varphi_0} = 0$ one can derive the dispersion relation, i.e. the dependence of soliton velocity v_s on its amplitude φ_0:

$$\varphi_0 \frac{\pi^{1/2}}{2} = \int_{-\infty}^{\infty} dt (t - v_s)^2 \exp(-t^2) \times \left[\left[1 + \frac{\varphi_0}{(t - v_s)^2} \right]^{1/2} - 1 \right]. \quad (16)$$

In the case of large amplitudes one can derive approximately

$$\frac{(\varphi_0 \pi)^{1/2}}{2} \simeq \int_{-\infty}^{\infty} dt \mid t \mid \exp[-(t + v_s)^2]. \quad (17)$$

Then we obtain:

$$0.25 \left(\frac{\pi}{\varphi_0} \right)^{1/2} \frac{d\varphi_0}{dv_s} = -2v_s \int_0^{\infty} dt t [\exp(-(t + v_s)^2) + \exp(-(t - v_s)^2)] + 2 \int_0^{\infty} dt t^2 [\exp(-(t - v_s)^2) - \exp(-(t + v_s)^2)]. \quad (18)$$

Using the approximate relation:

$$\frac{(\varphi_0 \pi)^{1/2}}{2} \simeq \int_{-\infty}^{\infty} dt \mid t \mid \exp[-(t + v_s)^2], \quad (19)$$

we obtain the inverse derivative of soliton velocity on its amplitude:

$$0.25 (\pi/\varphi_0)^{1/2} \frac{d\varphi_0}{dv_s} = -v_s (\pi \varphi_0)^{1/2} + 2 \int_0^{\infty} dt t^2 [\exp(-(t - v_s)^2) - \exp(-(t + v_s)^2)] \simeq v_s [-(\pi \varphi_0)^{1/2} + \exp(-v_s^2)[4 + 2(1 + 2v_s^2)]]. \quad (20)$$

6. THE DEPENDENCE OF SOLITON WIDTH ON AMPLITUDE AT LARGE ITS VALUES

From Poisson equation we derive:

$$\frac{(\partial_\eta \varphi)^2}{2} = \varphi_0 - \varphi - \frac{2}{\pi^{1/2}} \int_{-\infty}^{\infty} dt (t - v_s)^2 \exp(-t^2) \times \left[\left[1 + \frac{\varphi_0}{(t - v_s)^2} \right]^{1/2} - \left[1 + \frac{\varphi}{(t - v_s)^2} \right]^{1/2} \right]. \quad (21)$$

$$\frac{[\partial_\eta \varphi \mid_{\varphi = \varphi_0/2}]^2}{2} = \frac{\varphi_0}{2} - \quad (22)$$

$$- \frac{2\varphi_0^{1/2}}{\pi^{1/2}} \int_{-\infty}^{\infty} dt \mid t \mid \left(1 - \frac{1}{2^{1/2}} \right) \exp[-(t + v_s)^2].$$

We use approximate relation:

$$\frac{(\varphi_0 \pi)^{1/2}}{2} \simeq \int_{-\infty}^{\infty} dt \mid t \mid \exp[-(t + v_s)^2]. \quad (23)$$

Then we derive:

$$\frac{[\partial_\eta \varphi \mid_{\varphi = \varphi_0/2}]^2}{2} \simeq \left(\frac{\varphi_0}{2} \right) (2^{1/2} - 1). \quad (24)$$

From the latter we obtain approximately the width $\Delta\eta$ of soliton:

$$\Delta\eta = \frac{\varphi_0}{\partial_\eta \varphi \mid_{\varphi = \varphi_0/2}} \simeq \left[\frac{\varphi_0}{(2^{1/2} - 1)} \right]^{1/2}. \quad (25)$$

The soliton width increases with growth φ_0. This has been observed at numerical simulation and in experiment. Because the soliton width increases with amplitude growth, it is interesting to consider the role of trapped electrons.

7. THE ROLE OF TRAPPED ELECTRONS

In approximation of small amplitude the density of trapped electrons equals

$$2 \exp \left(\frac{e\varphi}{T_{tr}} \right) \int_0^A du f_{tr}(u) \simeq \quad (26)$$

$$\simeq \left(\frac{2n_{tr}}{\pi^{1/2}} \right) \left[\left(\frac{e\varphi}{\mid T_{tr} \mid} \right)^{1/2} - \left(\frac{e\varphi}{\mid T_{tr} \mid} \right)^{3/2} \frac{2}{3} \right].$$

Comparing this expression with density of untrapped electrons, one can see that the nonlinearity of trapped electrons is stronger and has the same sign that the nonlinearity, determined by untrapped electrons. Thus if the density of trapped electrons is essential, then they determine the properties of solitary hump of electrical potential. The trapped electrons are slowed down on the ends of the soliton (on its periphery, where $\varphi \simeq 0$). There their density

$$2n_{tr} \exp\left(\frac{e\varphi}{T_{tr}}\right) \int\limits_0^A du f_{tr}(u)$$

increases. Hence it is easy for trapped electrons to provide necessary for soliton the spatial distribution of electron density perturbation δn_e (humps of δn_e near the ends of the soliton (on its periphery)).

Also it is easy for trapped electrons to provide necessary for soliton the dip δn_e in its centre, if the trapped electrons are located near separatrix, i.e. at $T_{tr} \leq 0$. In other words the soliton is easier to form, if it forms hole in electron phase space. If the trapped electrons are located near separatrix, i.e. $T_{tr} \leq 0$, $| T_{tr} | \gg T_e$, then the density of the trapped electrons decreases quickly inside the soliton and it is essential only at $\varphi \ll \varphi_0$. Then we derive approximately:

$$n_{tre} = 2n_{tr} \exp\left(\frac{e\varphi}{T_{tr}}\right) \int\limits_0^A du f_{tr}(u) \simeq$$
$$\simeq \left[\frac{2n_{tr}}{V_{tr}(2\pi)^{1/2}}\right] \left(\frac{e\varphi}{m}\right)^{1/2} \exp\left(\frac{e\varphi}{T_{tr}}\right). \quad (27)$$

n_{tre} decreases inside the soliton ($\varphi \to \varphi_0$) as well as outside the soliton ($\varphi \to 0$). The density n_{tre} is essential only near the points of reflection of the trapped electrons (on its periphery).

8. CONCLUSIONS

The excitation of soliton by nonstationary electrical field has been described analytically for the first time. The properties of soliton have been described in strongly nonlinear case. The dependences of width and velocity of soliton on its amplitude at large values of amplitude have been derived. It has been shown that with amplitude growth the width of soliton increases at large values of amplitude both in experiments and numerical simulation.

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ВОЗБУЖДЕНИЕ И СВОЙСТВА СОЛИТОНА БОЛЬШОЙ АМПЛИТУДЫ ВБЛИЗИ ФОЛЬГИ ПРИ ВЗАИМОДЕЙСТВИИ С НЕЙ ЛАЗЕРНОГО ИМПУЛЬСА

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Описаны свойства и возбуждение наблюдаемого в экспериментах солитонного горба электрического потенциала большой амплитуды, двигающегося с тепловой скоростью электронов плазмы, вблизи фольги при взаимодействии с ней лазерного импульса. Этот солитон наблюдался при взаимодействии с фольгой толщиной 26 микрон, лазерного импульса длительностью 1 нс и мощностью 10^{14} Вт/см^2. Впервые описано возбуждение солитона нестационарным электрическим полем. Описаны свойства солитона в сильно нелинейном случае. Получена зависимость ширины и скорости солитона от его амплитуды при больших ее значениях. Показано, что с ростом амплитуды ширина солитона растет при больших значениях амплитуды, как в экспериментах и численном моделировании.

ЗБУДЖЕННЯ І ВЛАСТИВОСТІ СОЛІТОНА ЗНАЧНОЇ АМПЛІТУДИ ПОБЛИЗУ ФОЛЬГИ ПРИ ВЗАЄМОДІЇ З НЕЮ ЛАЗЕРНОГО ІМПУЛЬСУ

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Описані властивості і збудження солітонного горба електричного потенціалу великої амплітуди, який спостерігався в експериментах і рухався з тепловою швидкістю електронів плазми, поблизу фольги при взаємодії з нею лазерного імпульсу. Цей солітон спостерігався при взаємодії з фольгою товщиною 26 мікрон лазерного імпульсу тривалістю 1 нс і потужністю 10^{14} Вт/см^2. Вперше описано збудження солітона нестаціонарним електричним полем. Описані властивості солітона в дуже нелінійному випадку. Отримана залежність ширини та швидкості солітона від його амплітуди при великих її значеннях. Показано, що зі збільшенням амплітуди ширина солітона росте при великих амплітудах, як в експериментах та числовому моделюванні.