THE ABSORPTION OF THE VARIABLE ELECTRIC FIELD IN SUPERFLUID HELIUM BY THE AKHIEZER MECHANISM

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Earlier a series of experiments showed that the energy of a roton in superfluid helium is linearly dependent on electric field. A variable adiabatically changing external electric field leads to a modulation of the roton energy and the absorption of the field by the Akhiezer mechanism. We calculate the absorption of energy, which is resonant at the frequencies of the second-sound waves.

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1. INTRODUCTION

In [1] A.I. Akhiezer proposed a new mechanism for low frequency sound absorption in solids, which later was named as Akhiezer mechanism. Its basic idea is that the sound wave plays the role of the external force, adiabatically modulating the energy of thermal phonons and leading to a deviation of the phonon distribution function from equilibrium. Due to fast phonon-phonon interactions, a new quasilocal distribution function is installing. The entropy of the system increases in the process of relaxation, which leads to heating of the phonons gas, i.e to the absorption of sound waves. Further, this mechanism has been used to describe the absorption of sound in metals [2], ferrodielectrics [3], as well as absorption of a variable magnetic field in ferromagnetic by magnon gas [4]. A new method for adiabatic magnetic pumping of energy into the weakly turbulent plasma was offered in [5], based on this mechanism. The general theory of adiabatic heating of a quasi-particles gas by external fields, which modulates the energy of quasiparticles has been presented in [6].

It was also shown that if there are weakly damped waves in a quasiparticles gas, such as second-sound waves, with frequency Ω_{II} and damping factor Γ_{II} ($\Omega_{II} \ll \Gamma_{II}$), then the energy absorption efficiency of the external fields at resonance $\Omega=\Omega_{II}$ increases in $\left(\frac{\Omega_{II}}{\Gamma_{II}}\right)^2$ times. On the other hand, this means that there is a resonant excitation of second sound waves (SSW) in a quasiparticles gas by external fields [7, 8].

There was observed an occurrence of electrical induction in Rybalko experiments on the second sound excitation [9] in superfluid helium by a variable heat flux. In addition, there were also carried out experiments on the excitation of SSW by variable electric field, when its frequency coincides with the frequency of second sound. We believe that the resonance absorption of an variable electric field, which was observed in these experiments, takes place by the Akhiezer mechanism.

2. ADIABATIC HEATING OF THE QUASIPARTICLES GAS

Spatial and temporal dependence of the energy ε is conditioned by the dependence of the medium parameters on the variables of external fields. These options, which can be scalars, vectors, tensors, or functionals of certain variables are denoted by $\widehat{A}$. This character will serve to distinguish different operations with the corresponding quantities.

The value $\widehat{A}$ can be represented as:

$$\widehat{A} = \widehat{A}_0 + \delta\widehat{A}, \qquad (1)$$

where $\widehat{A}_0$ is the value $\widehat{A}$ in the absence of temporary external fields, and $\delta\widehat{A}$ is the variation of $\widehat{A}$ value by variable external fields. This representation is valid in conditions of weak spatial inhomogeneity of the external fields (i.e. when the inhomogeneity scale L_c is considerably larger than the characteristic wavelength of quasiparticles λ) and the slow time variation of the external fields (i.e., when the characteristic times of the external fields t_c is much larger than the characteristic period T of the quasiparticles oscillation)

$$L_c \gg \lambda, \quad t_c \gg T. \qquad (2)$$

This leads to an explicit dependence of the quasiparticle energy on the spatial ($\vec{r}$) and temporal (t) variables. Conditions (2) are the conditions of adiabatic change of the medium parameters. Using this notation, we write down the quasiparticle energy of j species with momentum $\vec{p}$ with a small adiabatic change of parameters on which it depends, in the form [10]:

$$\varepsilon^{(j)}\left(\widehat{A},\vec{p}\right) = \varepsilon_0^{(j)}\left(\widehat{A}_0,\vec{p}\right)\left(1+\widehat{a}^{(j)}\right), \qquad (3)$$

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where $\widehat{a}^{(j)} \equiv \widehat{a}^{(j)}(\vec{p},\vec{r},t)$ is the depth of the quasiparticle energy modulation by external fields, which is equal

$$\widehat{a}^{(j)} = \left(\frac{\partial}{\partial \widehat{A}} \ln \varepsilon\right)_{\widehat{A}=\widehat{A}_0} \delta\widehat{A}. \qquad (4)$$

Conditions of adiabaticity of parameter changes mean that

$$\left|\tau^{(j)} \frac{d}{dt} \ln \left|\widehat{a}^{(j)}\right|\right| \ll 1,$$
$$\left|\tau^{(j)} \frac{d\varepsilon_0^{(j)}}{d\vec{p}} \cdot \nabla \ln \left|\widehat{a}^{(j)}\right|\right| \ll 1, \qquad (5)$$

where $\tau^{(j)}$ is the lifetime of the quasiparticles.

In [6], based on the Cauchy-Bunyakovskiy inequality, the following formula for the estimated growth rate of entropy was obtained:

$$(\dot{S})_{st} = \delta Q \geq \left\langle \frac{|\delta_p|^2}{\sum_n n\nu_p^{(n)}} \right\rangle. \qquad (6)$$

Here $\langle \ldots \rangle$ means the summation, $\langle f \rangle = \sum_j \langle f \rangle_j$; $\langle f \rangle_j = \sum_{\vec{p}} f^{(j)} N_0^{(j)}(1+N_0^{(j)})$; δ_p is the kinematical part of the linearized Boltzmann equation for the quasiparticle distribution function [10]:

$$\delta_p = \left(\frac{\partial}{\partial t} + \vec{g}\frac{\partial}{\partial \vec{r}} - \frac{\partial \varepsilon_p}{\partial \vec{r}}\frac{\partial}{\partial \vec{p}}\right) \frac{\varepsilon(\vec{p}) - (\vec{p}\vec{V}_n)}{T_0(1+\vartheta)}, \qquad (7)$$

$\nu_p = \frac{1}{\tau_p}$ is the collision frequency; $N_0 = \left(\exp \frac{\varepsilon(\vec{p})-(\vec{p}\vec{V}_n)}{T_0(1+\vartheta)} - 1\right)^{-1}$ is the local equilibrium distribution function; it is established by rapid interaction of quasiparticles with each other; $\vec{V}$, ϑ are parameters, which define it, and T_0 is the thermodynamic equilibrium temperature.

To find the energy absorbed per unit time the dissipation function $D = T\dot{S}$ should be governed by quasi-gas-dynamics equations, taking into account external fields to express the values of thermodynamic parameters of the exciting force through external sources.

3. ENERGY ABSORPTION OF VARIABLE ELECTRIC FIELD BY ROTONS GAS IN SUPERFLUID HELIUM

The excited state of He II is described by a gas of quasiparticles (phonons, rotons). Experiments [9] indicate that the electric field vector $\vec{E}$ and the electric induction $\vec{D}$ (polarization vector) are a set of thermodynamic parameters, which characterizes the properties of superfluid helium. Therefore it is natural to assume that the quasiparticle energy depends linearly on the electric field [11], which is supported by experimental data [12]. Quasiparticle momentum $\vec{p}$ is the single vector that characterizes the quasiparticle. That is why the electric field vector must be multiplied by it to form a scalar. From these considerations the energy of quasiparticles in superfluid He II in the presence of motion with a velocity $\vec{V}_s$ can be written as follows [11]:

$$\varepsilon(\vec{p},\vec{E}) = \varepsilon(\vec{p}) + \vec{p}(\alpha\vec{E} + \vec{V}_s). \qquad (8)$$

where V_s is the velocity of superfluid flow; α is a phenomenological parameter. To satisfy the invariance of the quasiparticle energy with respect to time reversal it is necessary to satisfy the condition $\alpha(-t) = -\alpha(t)$ for the parameter α. In [11] it was shown that this condition is satisfied. In addition, in [13] a dipole moment of a roton was calculated, which is proportional to the roton momentum.

Note that in this paper, we do not assume the presence of spontaneous polarization of the liquid in the superfluid state. However, the terms containing the electric field in (8) are the energy of a dipole in an external field, which may indicate the appearance of polarization per unit volume of superfluid. However, to determine the causes of this situation, a theory of this effect in a microscopic approach is required that goes beyond the scope of this paper.

We apply the general provisions of Section 2 to the case of superfluid helium in the temperature range 1.4-2 K, in which rotons play a major role in determining the thermodynamic and kinetic properties. In the case of external field the mean velocity of the superfluid motion $\vec{V}_s$ is an external parameter and external AC electric field

$$\vec{E} = \vec{E}_0 e^{i(\vec{q}\vec{r}-\Omega t)} \qquad (9)$$

play the role of exciting force. The modulation depth of the roton energy is determined by the exciting force and equals $\hat{a} = \frac{\alpha(\vec{p}\vec{E})}{\varepsilon_r}$. Value δ_p will be

$$\delta_p = \frac{-i\Omega}{T_0}(\vec{p}(\vec{V}_n - \vec{V}_s - \alpha\vec{E}) + \varepsilon_0\vartheta - \frac{(\vec{q}\vec{g})}{\Omega}(\varepsilon_0\vartheta + \vec{p}\vec{V}_n)),$$

where $\varepsilon_0 = \Delta + \frac{(p-p_0)^2}{2\mu}$ is the roton energy.

In [11] the equations of two-fluid hydrodynamics with electric fields was obtained. Considering the magnitude ϑ, $\vec{V}_n$ and $\vec{V}_s$ as small values we find the following system of linearized equations of two-fluid hydrodynamics

$$C\frac{\partial \vartheta}{\partial t} + \bar{S}\mathrm{div}\vec{V}_n = 0, \qquad (10)$$
$$\frac{\partial \vec{V}_n}{\partial t} - \frac{\partial \vec{V}_s}{\partial t} - \alpha\frac{\partial \vec{E}}{\partial t} + \frac{T\bar{S}}{\rho_n}\nabla\vartheta = 0, \qquad (11)$$
$$\frac{\partial \vec{V}_s}{\partial t} = -\nabla\mu,$$

where $d\mu = -\frac{1}{\rho}d\bar{S}$ (taking into account the smallness of the coefficient of thermal expansion).

Differentiating (10) and substituting in (11) we obtain the equation of forced SSW

$$\frac{\partial^2 \vartheta}{\partial t^2} - \frac{T\bar{S}\rho_s}{\rho\rho_n C}\Delta\vartheta = -\frac{\bar{S}}{C}\alpha\nabla\frac{\partial \vec{E}}{\partial t}. \qquad (12)$$

Note that the electric field should be variable and spatially inhomogeneous. Assuming that the external force is (9), we obtain the following solution

$$\vartheta = \frac{\alpha \Omega \bar{S}(\vec{q}\vec{E})}{C\left[(\Omega^2 - q^2 W_{II}^2) - 2i\Omega_{II}\Gamma_{II}\right]} e^{-i(\Omega t - \vec{q}\vec{r})}, \quad (13)$$

taking into account the attenuation of SSW with coefficient Γ_{II}. In the absence of the electric field equations (10), (11) describe a weak damped SSW with a frequency $\Omega_{II} = qW_{II}$, where $W_{II}^2 = \frac{T\bar{S}\rho_s}{C\rho_n\rho}$; ρ_s and ρ_n are the densities of the superfluid and normal components; C is the density of the heat capacity; $\bar{S}$ is the entropy density of a roton gas. Substituting this solution in the dissipative function, we obtain the rate of energy absorption from external sources, which in the case of resonance with account of the main resonant terms has the form

$$D_{res} \sim \frac{\Delta^2}{T_0^2} \frac{\Omega_{II}^2}{\Gamma_{II}^2} \frac{\alpha^2 \rho \Omega_{II}^2}{\nu_r} \frac{\rho_n}{\rho_s C} N_r E^2. \quad (14)$$

4. CONCLUSIONS

It is shown that an additional process of energy absorption of the external variable electric field in the superfluid helium is realized by Akhiezer mechanism. In the resonant case, when the frequency of the electric field coincides with the roton frequency, the effective absorption increases at a $\frac{\Omega_{II}^2}{\Gamma_{II}^2}$ time. In this case a second sound wave is excited by external electric fields, which was observed experimentally in [9]. Knowing from the experiment the rate of energy absorption at resonance, we can determine the value of the phenomenological parameter α.

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ПОГЛОЩЕНИЕ ПЕРЕМЕННОГО ЭЛЕКТРИЧЕСКОГО ПОЛЯ В СВЕРХТЕКУЧЕМ ГЕЛИИ ПО МЕХАНИЗМУ АХИЕЗЕРА

А.С. Наумовец, В.Д. Ходусов

Ранее экспериментально было установлено, что энергия ротона в сверхтекучем гелии линейно зависит от электрического поля. Адиабатически изменяющееся внешнее электрическое поле приводит к модуляции энергии ротона и поглощению энергии этого поля по механизму Ахиезера. Вычислено поглощение энергии, которое имеет резонансный характер на частотах волн второго звука.

ПОГЛИНАННЯ ЗМІННОГО ЕЛЕКТРИЧНОГО ПОЛЯ У НАДПЛИННОМУ ГЕЛІЇ ЗА МЕХАНІЗМОМ АХІЄЗЕРА

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Раніше експериментально було встановлено, що енергія ротонів у надплинному гелії лінійно залежить від електричного поля. Зовнішнє електричне поле, що змінюється адіабатично, призводить до модуляції енергії ротонів і поглинання енергії цього поля за механізмом Ахієзера. Обчислено поглинання енергії, яке має резонансний характер на частотах хвиль другого звуку.