NONLOCAL EFFECT INFLUENCE ON THE VORTEX FILAMENT STRUCTURE IN BOSE-GAS

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Using the Gross-Pitaevskii equation (GPE), which accounts for a weak non-locality effects, we study the structure of the vortex filaments in a Bose-gas. We show that the sign of the additional terms in the generalized GPE depends on the form of the interaction potential and can be positive or negative. The generalized GPE for a single vortex is solved numerically. It is demonstrated that the non-locality can cause either an increase or a decrease of the vortex core radius.

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1. INTRODUCTION

In order to study the properties of the superfluid Bose-gas of weakly interacting particles, the equation obtained by Gross and Pitaevskii [1, 2] is widely used (GPE). The characteristic distance, at which the macroscopic wavefunction of the condensate significantly varies, is the correlation length $\xi = \hbar/\sqrt{2mn_0U_0}$, where m is the particle mass, n_0 is the equilibrium density, and U_0 is the interaction strength. In the standard formulation of the GPE, it is assumed that $\xi \gg r_0$, where r_0 is the characteristic range of the inter-particle interaction, such that the effects due to finiteness of r_0 are not accounted for. A particular solution of the GPE is the single vortex filament solution. In a dimensionless form, the equation for the vortex filament is universal and does not depend on any parameters, which could be specific to a system [3]. In the current paper the structure of the vortex filament is studied by means of a modified GPE, which accounts for a small but finite value of the ratio r_0/ξ. In this case the dimensionless form of the GPE contains a dimensionless parameter $\alpha \sim (r_0/\xi)^2$ and, therefore, the modified GPE is not universal. The parameter α is calculated for a repulsive potential of a "semi-transparent sphere" type with an account for the Van-der-Vaals attraction at long distance. By means of a numerical solution of the modified GPE we have shown that for positive α the radius of the vortex core increases and for negative α it decreases.

2. NONLOCAL AND MODIFIED FORM OF GROSS-PITAEVSKII EQUATION

For arbitrary relation between the correlation length and the inter-particle interaction range the dynamical GPE for the macroscopic wavefunction $\Phi(\vec{r},t)$ is a integro-differential equation [4]:

$$i\hbar\frac{\partial\Phi(\vec{r},t)}{\partial t} = -\left(\frac{\hbar^2}{2m}\Delta + \mu\right)\Phi(\vec{r},t) + \Phi(\vec{r},t)\int d\vec{r}'\,U(\vec{r}-\vec{r}')\left|\Phi(\vec{r}',t)\right|^2, \quad (1)$$

where μ is the chemical potential and $U(\vec{r}-\vec{r}')$ is interparticle potential.

Equation (1), in particular, can be used for description of such the short-wave excitations of a condensate, that their wavelength is of the order of the inter-particle distance. In Ref. [4] it was shown that for $\xi \sim r_0$ the short-wave excitation spectrum has a minimum at a given value of the momentum, $p_0 \sim \hbar/r_0$, which is analogous to the roton minimum of the superfluid helium. For small values of the ratio r_0/ξ one can reduce the integro-differential equation (1) to a differential equation, which accounts for small non-locality effects. In order to do that, let us assume that the inter-particle potential depends only on the inter-particle distance and can be written in the form

$$U(|\vec{r}-\vec{r}'|) = U_0\delta(\vec{r}-\vec{r}') + u(|\vec{r}-\vec{r}'|), \quad (2)$$

where

$$U_0 = \int d\vec{r}\,U(\vec{r}). \quad (3)$$

From Eq. (3) one obtains

$$\int d\vec{r'}\,u(|\vec{r}-\vec{r}'|) = 0. \quad (4)$$

The nonlocal effect is defined by the function $u(|\vec{r}-\vec{r}'|)$. If one neglects the contribution of this

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function, the standard form of GPE [1, 3] is recovered. After substitution of (2) into (1), one obtains the GPE in the form

$$i\hbar\frac{\partial\Phi(\vec{r},t)}{\partial t}=-\left(\frac{\hbar^2}{2m}\Delta+\mu\right)\Phi(\vec{r},t)$$
$$+\Phi(\vec{r},t)\,U_0\,|\Phi(\vec{r},t)|^2+\Phi(\vec{r},t)F(\vec{r},t), \quad (5)$$

where

$$F(\vec{r},t)\equiv\int d\vec{r}'\,u(\vec{r}')\,|\Phi(\vec{r}-\vec{r}',t)|^2\,. \quad (6)$$

Expanding the function $\Phi(\vec{r}-\vec{r}')$ of (6) in series around $\vec{r}$ up to quadratic terms, one obtains the equation

$$i\hbar\frac{\partial\Phi(\vec{r},t)}{\partial t}=-\left(\frac{\hbar^2}{2m}\Delta+\mu\right)\Phi(\vec{r},t)$$
$$+\Phi(\vec{r},t)\,U_0\,|\Phi(\vec{r},t)|^2+\frac{A}{2}\Phi(\vec{r},t)\Delta\,|\Phi(\vec{r},t)|^2\,, \quad (7)$$

where the constant A is defined as

$$A=\frac{1}{3}\int d\vec{r}\vec{r}^2u(|\vec{r}|). \quad (8)$$

Chemical potential in Eq. (7) can be expressed via the equilibrium density $\mu=n_0U_0$, where $n_0=|\Phi_0(\vec{r})|^2$. Notice, that the equation in the form (7) was used in Ref. [5] in order to study the two-dimensional solitons. In Ref. [6] an analogous equation was used in the stability analysys of a system with respect to a collapse (however, from the beginning, only the attractive forces are considered, i.e., $U_0<0$). In what follows, in order to study the vortex structure, we will use the stationary equation in the form

$$\frac{\hbar^2}{2m}\Delta\Phi(\vec{r})+\Phi(\vec{r})\,U_0\left(n_0-|\Phi(\vec{r})|^2\right)$$
$$-\frac{A}{2}\Phi(\vec{r})\Delta\,|\Phi(\vec{r})|^2=0. \quad (9)$$

Let us assume that the potential $U(r)$ is of a “semi-transparent sphere” type at a distance of $r<\ r_0$, but for $r>r_0$ it accounts for a Van-der-Vaals attraction:

$$U(\mathbf{r})=\begin{cases}U_{max}, & r\le a,\\ -C/r^6, & r>a,\end{cases} \quad (10)$$

where C is positive. According to Eq. (8)

$$A=\frac{4\pi}{15}U_mr_0^5\left(1-\frac{5C_0}{U_mr_0^6}\right). \quad (11)$$

For the potential (10) one has

$$U_0=\frac{4\pi U_mr_0^3}{3}\left(1-\frac{C_0}{U_mr_0^6}\right). \quad (12)$$

We assume that the parameter U_0 is positive, and therefore the condition $C_0<U_mr_0^6$ is satisfied. Thus, for $0<C_0<U_mr_0^6/5$ in eqs. (7), (9) the parameter $A>0$, but for $U_mr_0^6/5<C_0<U_mr_0^6$ one has $A<0$. In both cases the condition $U_0>0$ holds true.

Notice that the small non-locality due to Eq. (7) leads to a small modification of the Bogolyubov spectrum [7] of the small oscillations of the condensate

$$\hbar^2\omega^2=\epsilon_k\left(\epsilon_k-n_0Ak^2\right)+2n_0U_0\epsilon_k, \quad (13)$$

where $\epsilon_k=\hbar^2k^2/2m$, k is the wave vector and ω is the frequency of condensate oscillations.

3. NUMERICAL SOLUTION OF WEAK NONLOCAL GP EQUATION

For numerical analysis of the vortex solutions of the Eq. (9) it is convenient to write it down in terms of dimensionless variables

$$\tilde{\vec{r}}=\vec{r}/\xi\,,\quad \psi=\phi/\sqrt{n_0}.$$

Thus we obtain

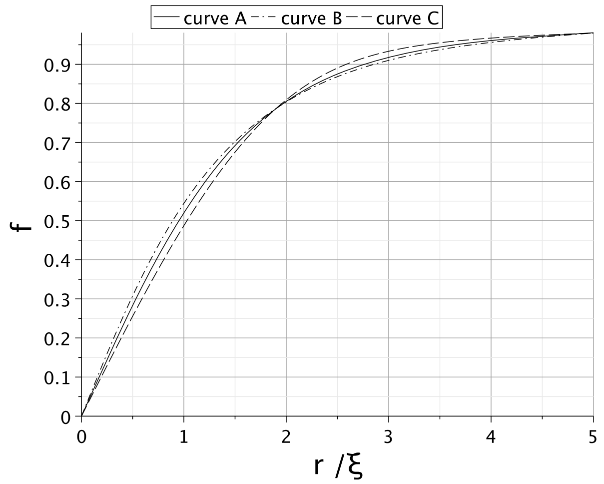
$$\tilde{\Delta}\psi+\left(1-|\psi|^2\right)\psi-\alpha\psi\tilde{\Delta}\,|\psi|^2=0. \quad (14)$$

One can see that the only dimensionless parameter in Eq. (14) is $\alpha=A/2U_0\xi^2$ and it is a measure of non-locality. One can assume that this parameter is small such that $|\alpha|<<1$. In order to find a vortex solution, consider an ansatz for the wavefunction $\psi=fe^{in\varphi}$, where n is a natural number and write down Eq. (14) in cylindrical coordinates

$$f''+\frac{1}{\tilde{r}}f'+\left(1-\frac{n^2}{\tilde{r}^2}\right)f-f^3 \quad (15)$$
$$-2\alpha f[ff''+\frac{1}{\tilde{r}}f^2+f'^2]=0.$$

Here we introduced the next generally accepted definition $f'=df/d\tilde{r}$.

Numerical solutions of the Eq. (15) for $n=1$ with positive and negative values of the non-locality parameter α and in the case $\alpha=0$ (local interaction) are given in the figure. We see that for large distances in all three cases the solutions coincide. They have the same asymptotics, which is equal to 1. Distinction in behavior of solutions is exhibited on the distances compared with the correlation length $r\sim\xi$. From the figure it follows also that the vortex core radius gets enhanced for positive α, when the inter-particle repulsion dominates. In case when the attraction is significant(the case $\alpha<0$), the vortex core radius gets decreased, i.e., the dominance of the inter-particle attraction leads to the vortex shrinking. In the considered cases the correlation length $\xi=\hbar/\sqrt{2mn_0U_0}$ defines the order of the magnitude of the vortex core radius [3].



Solution of Eq. (15) in the cases of local interaction, $\alpha = 0$ (curve A); interaction defined by Eq. (10), $\alpha = -0.45$ (curve B); interaction defined by Eq. (10), $\alpha = 0.45$ (curve C)

4. CONCLUSIONS

By means of a modified GPE, which accounts for small non-locality effects due to the finiteness of the inter-particle interaction range, we have studied the structure of the vortex filament in a superfluid Bose-gas. It was shown that the size of the vortex core depends on the sign of the parameter, which describes the non-locality. For positive values of this parameter, the vortex core radius gets increased and the negative ones lead to the vortex shrinking.

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ВЛИЯНИЕ ЭФФЕКТА НЕЛОКАЛЬНОСТИ НА СТРУКТУРУ ВИХРЕВОЙ НИТИ В БОЗЕ-ГАЗЕ

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С помощью уравнения Гросса-Питаевского (ГП), учитывающего эффект слабой нелокальности, исследована структура вихревой нити в бозе-газе. Показано, что знак постоянной перед дополнительным слагаемым в обобщенном уравнении ГП зависит от вида потенциала взаимодействия и может быть как положительным, так и отрицательным. Обобщенное уравнение ГП для одиночного вихря решено численно. Показано, что учет эффекта нелокальности может приводить как к увеличению, так и к уменьшению радиуса сердцевины вихря.

ВПЛИВ ЕФЕКТА НЕЛОКАЛЬНОСТІ НА СТРУКТУРУ ВИХРОВОЇ НИТКИ В БОЗЕ-ГАЗІ

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За допомогою рівняння Гросса-Питаєвського (ГП), яке враховує ефект малої нелокальності, досліджена структура вихру у надплинному бозе-конденсаті. Показано, що знак сталої перед додатковим доданком в узагальненому рівнянні ГП залежить від потенціалу взаємодії і може бути як додатнім, так і від’ємним. Узагальнене рівняння ГП для одинарного вихру розв’язано чисельно. Показано, що врахування ефекта нелокальності може привести як до збільшення, так і до зменшення радіусу серцевини вихру.