PHONONS, ROTONS AND RIPPLONS AT INTERFACES

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We use dispersive hydrodynamics to describe thermal excitations of superfluid helium 4. Dispersion relation of the bulk quasiparticles, phonons and rotons, acts as an input parameter of the theory. Wiener and Hopf method is used to solve nonlocal equations of the fluid in half-space. The dispersion relation helium surface excitations, ripplons, is derived and analyzed; numerical solution reveals its new unusual branch. The same method applies to the description of bulk quasiparticles' interaction with the interface with a solid. All quasiparticles creation probabilities are derived and weak interaction of rotons with negative dispersion with interfaces is explained.

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1. INTRODUCTION

The bulk and surface thermal excitations of superfluid helium are well-defined and long-lived both for small wave vectors and for large, comparable with inverse average atomic separation. The distinctive dispersion relation of the bulk excitations, shown in Fig. 1, is measured in experiments on neutron scattering [1]. There is an almost linear phonon part, and a parabola-like roton part. Rotons with negative group velocity, to the left from the minimum, are called R^- rotons, and with positive group velocity R^+ rotons.

The bulk excitations were studied in a large series of experiments on pulses of quasiparticles in helium at very low temperatures (see [3, 4]). R^- rotons, however, eluded direct detection, supposedly due to anomalously weak interaction with interfaces, as solid heaters and bolometers were usually used for creation and detection of pulses. This was further affirmed by their eventual detection in [3] by means of a special source and quantum evaporation, and the effect needed theoretical description.

Ripplons are quantised capillary waves on the surface of He II. Experiments on neutron scattering in thin films [5] showed, that they are well-defined quasiparticles even at frequencies of the order of the roton gap. It was important to derive theoretically their dispersion relation for large wave vectors (see [6]).

Both problems were solved using a model in which the quantum fluid is described as a continuous medium at all length scales, down to the interatomic distances [7, 8]. This approach naturally allows to consider a quantum fluid with essentially arbitrary nonlinear dispersion relation, and the two problems turned out to be two sides of one. In this paper we give a short review of the model and method used, and the main results of this approach.

2. DISPERSIVE HYDRODYNAMICS

In a quantum fluid atoms are delocalized and values of variables of continuous medium velocity $\mathbf{v}$, density ρ and pressure P can be introduced in every mathematical point in space. Then it can be described [7] by the linearized equations of ideal liquid with equilibrium density ρ_q

$$\dot{\mathbf{v}} = -\rho_q^{-1}\,\nabla P, \quad \dot{\rho} = -\rho_q \nabla\mathbf{v}, \tag{1}$$

complemented with a nonlocal equation of state

$$\rho(\mathbf{r}) = \int\limits_V d^3r'\, h(\mathbf{r},\mathbf{r}')P(\mathbf{r}'). \tag{2}$$

The kernel h can only be the function of $|\mathbf{r}-\mathbf{r}'|$ for a homogeneous and isotropic fluid; area of integration V is the volume filled with the fluid. In terms of one variable, for example pressure P, this system leads to a nonlocal wave equation of the form

$$\Delta P = \int d^3r'\, h\big(|\mathbf{r}-\mathbf{r}'|\big)\,\ddot{P}(\mathbf{r}'). \tag{3}$$

If the fluid fills the infinite space, the integrand is a convolution, and Fourier transform gives us the relation between the kernel's Fourier image $h(k)$ and the dispersion relation of the medium $\Omega(k)$:

$$h(k) = \frac{k^2}{\Omega^2(k)}. \tag{4}$$

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3. FLUID IN HALF-SPACE

When the fluid fills a half-space $z > 0$, equation for P takes the form

$$\Delta P = \int\limits_{z_1>0} d^3r'\, h\big(|\mathbf{r}-\mathbf{r}'|\big)\ddot{P}(\mathbf{r}'), \qquad (5)$$

so there is no convolution any more. Equations of this type can be solved by the Wiener and Hopf method, in which the equation is reduced to a Hilbert-Riemann boundary problem (see [9]), in our case

$$P^-(k_z;\omega,\mathbf{k}_\tau)J(k_z;\omega,\mathbf{k}_\tau) + P^+(k_z;\omega,\mathbf{k}_\tau) = 0 \qquad (6)$$
$$\text{for} \quad k_z \in (-\infty,\infty),$$

$$\text{where} \quad J(k_z;\omega,\mathbf{k}_\tau) = \frac{\Omega^2(k)-\omega^2}{\Omega^2(k)}. \qquad (7)$$

Here k_z and $\mathbf{k}_\tau$ are normal and tangential to the interface components of wave vector $\mathbf{k} = k_z\mathbf{e}_z + \mathbf{k}_\tau$; $P^+(k_z)$ and $P^-(k_z)$ are two functions to be found, analytic in the upper half-plane $\mathcal{C}_+$ and lower half-plane $\mathcal{C}_-$ of the complex variable k_z correspondingly, so that the signs in the superscripts denote their domains of analyticity. The relation (6) is between their limiting values on the real line of k_z, which separates those domains $\mathcal{C}_\pm$. $P(\mathbf{r})$ at $z > 0$ is obtained as the inverse Fourier transform of $P^-(\mathbf{k})$.

The general solution for $P^\pm$, that takes into account singular behavior of J at $k = 0$ and asymptotic boundary conditions for P^- at $z \to \infty$, utilizes factorization of an auxiliary density

$$I(k_z;\omega,\mathbf{k}_\tau) = J(k_z;\omega,\mathbf{k}_\tau)\frac{k^2}{k^2+A^2}, \qquad (8)$$

which is bounded and separated from zero on $k \in \Re$ (here A is some arbitrary positive constant), i.e. its expression in the form

$$I(k_z) = \frac{I^-(k_z)}{I^+(k_z)}, \qquad (9)$$

where $I^-(k_z)$ is a function analytic and with no zeros in $\mathcal{C}_-$, and I^+ likewise in $\mathcal{C}_+$:

$$P^-(k_z;\omega,\mathbf{k}_\tau) = \frac{\text{const}}{(k-iA)I^-(k_z;\omega,\mathbf{k}_\tau)}. \qquad (10)$$

Solution to the factorization problem is in the general case given by

$$I^- = \exp\Big\{\frac{1}{2\pi i}\int\limits_{-\infty}^{\infty}\frac{dk'_z}{k'_z-k_z}\ln I(k'_z)\Big\}. \qquad (11)$$

It can be essentially simplified, however, if we assume the function Ω^2 is a polynomial of k^2 (and thus a polynomial of k_z^2). In this case the density is factorized in an obvious way in terms of all the roots $k_{i\,z}$ of polynomial equation

$$\Omega^2(k_z,\mathbf{k}_\tau) = \omega^2 : \qquad (12)$$

we obtain I^- (I^+) by collecting all the factors of I that have poles and zeros in $\mathcal{C}_+$ ($\mathcal{C}_-$). The solution is then brought to

$$P^-(k_z;\omega,\mathbf{k}_\tau) = C(\omega,\mathbf{k}_\tau)\cdot\tilde{P}(k_z;\omega,\mathbf{k}_\tau), \quad \text{where}$$
$$\tilde{P}(k_z;\omega,\mathbf{k}_\tau) = \frac{\prod\limits_{i\neq 1}\big[k_z - k_{i\,z}(0,\mathbf{k}_\tau)\big]}{\prod\limits_{i}\big[k_z - k_{i\,z}(\omega,\mathbf{k}_\tau)\big]}, \qquad (13)$$

and the products are taken over all the roots $k_{i\,z}$ in $\mathcal{C}_+$. The product in the numerator misses the one factor with $k_{i\,z}$ which turns to zero when $k_\tau = 0$, and is eliminated from I before factorization (8). This root, which corresponds to the long-wavelength phonon branch, we denote by subscript 1.

In the limit $\omega \to 0$ all the factors in the numerator and denominator of $\tilde{P}$ are subtracted except for the extra one $(k_z - k_{1\,z})$ in the denominator, and we have

$$P^-(k_z) \approx \frac{\text{const}}{k_z - k_{1\,z}}, \quad \text{where} \quad k_{1\,z}^2 \approx \frac{\omega^2}{s^2} - k_\tau^2, \quad (14)$$

s is sound velocity in the limit of small ω. Then solution in the coordinate space with a given ω is

$$P(\mathbf{r},t) \sim e^{i\mathbf{k}_{1\,z}\mathbf{r}-i\omega t}, \quad \mathbf{k}_{1\,z} = k_{1\,z}\mathbf{e}_z + \mathbf{k}_\tau. \qquad (15)$$

In the general case the inverse Fourier transform gives monochromatic in ω solutions in the coordinate space of the form

$$P(\mathbf{r},t;\omega) = C\sum_{k_{i\,z}\in C_+} r_i e^{i\mathbf{k}_i\mathbf{r}-i\omega t}, \quad (16)$$

$$\text{where} \qquad \mathbf{k}_i = k_{i\,z}\mathbf{e}_z + \mathbf{k}_\tau, \qquad (17)$$

and r_i is the residue of $\tilde{P}$ (13) in $k_{i\,z}$. Using the equations of continuous medium (1), we obtain for velocity

$$\mathbf{v}(\mathbf{r},t;\omega) = \frac{C}{\rho_q\omega}\sum_{k_{i\,z}\in C_+}\mathbf{k}_i r_i e^{i\mathbf{k}_i\mathbf{r}-i\omega t}. \qquad (18)$$

Each real root of $k_{i\,z}$ gives a traveling wave in the sum, and each complex root gives a damped wave.

4. RIPPLONS

If we want to consider the surface mode of the superfluid, we should assume there are no real roots $k_{i\,z}$ of the equation (12). This happens when k_τ^2 is greater than all the real roots of $\Omega^2(k) = \omega^2$ with respect to k^2. Then all $k_{i\,z}$ are divided into complex conjugate pairs, and the solution (13) is unique.

It should be accompanied by the boundary conditions that take account surface tension σ on the free surface. Writing that pressure at the surface is equal to the Laplace pressure σ/R_{curv}, where R_{curv} is the curvature radius, for small deviations of the surface from equilibrium we get

$$P\Big|_{z=0} = \sigma k_{\tau^2}\cdot\frac{\mathrm{v}_z}{i\omega}\Big|_{z=0}. \qquad (19)$$

On the other side, the solution in the fluid (16,18) gives

$$\mathrm{v}_z|_{z=0} = \frac{1}{\rho_q\omega}\frac{\sum k_{i\,z} r_i}{\sum r_i}\cdot P|_{z=0}. \tag{20}$$

As $P^-(k_z)$ is by construction analytic in $\mathcal{C}_-$, the poles $k_{i\,z}$ constitute all of its poles in the finite part of the complex plane k_z. The sums in the numerator and denominator are equal to residues of $\tilde{P}$ and $k_z\tilde{P}$ at infinity correspondingly, and those can be calculated directly by expanding $\tilde{P}$ at $k_z \to \infty$:

$$\mathrm{v}_z|_{z=0} = \frac{1}{\rho_q\omega}\left\{k_{1\,z} + \sum_{i>1}\left[k_{i\,z} - k_{i\,z}(\omega=0)\right]\right\}\cdot P|_{z=0}. \tag{21}$$

Comparing (19) and (21), we obtain the equation of dispersion relation for the surface modes $\omega(k_\tau)$:

$$\omega^2 = \frac{\sigma k_\tau^2}{i\,\rho_0}\Big\{k_{1\,z}(\omega,k_\tau) + \sum_{i>1}\left[k_{i\,z}(\omega,k_\tau) - k_{i\,z}(0,k_\tau)\right]\Big\}. \tag{22}$$

In the limit $\omega \to 0$ the sum tends to zero and equation turns into

$$\omega^2 = \frac{\sigma}{\rho_0}k_\tau^3\sqrt{k_\tau^2 - \omega^2/s^2}. \tag{23}$$

This is also the form it takes when $\Omega(k)$ is linear. Neglecting compressibility, in the limit $s \to \infty$ we get the classic relation $\omega^2 \sim k_\tau^3$.

Expanding $k_{i\,z}$ in powers of ω and k_τ, and taking into account the asymptotic $\omega^2 \sim k_\tau^3$, we can obtain further corrections:

$$\omega^2(k_\tau) = \frac{\sigma}{\rho_q}\cdot k_\tau^3\Big\{1 - \frac{\sigma}{2\rho_q s^2}k_\tau + \frac{\sigma^2}{8\rho_q^2 s^4}k_\tau^2 + \tag{24}$$

$$- i\frac{\sigma}{\rho_q}\sum_{i>1}\frac{\partial k_{i\,z}(\omega,0)}{\partial(\omega^2)}\Big|_{\omega=0}\cdot\Big(k_\tau^2 - \frac{\sigma}{\rho_q s^2}k_\tau^3\Big) + O\big(k_\tau^4\big)\Big\}.$$

The two summands after unity in the braces take into account compressibility, and those after them represent contribution of non-phonon roots. They take into account non-linearity of $\Omega(k)$ and at small k give only small corrections.

The consideration above applies to arbitrary dispersion relation $\Omega(k)$ which has linear long-wavelength phonon part. We are now particularly interested in superfluid helium's non-monotonic dispersion, and in what happens to ripplon's dispersion relation close to the frequency of the roton gap Δ_{rot}. In this region dispersion of the two roton roots can be well approximated by a parabola

$$\Omega(k) \approx \Delta_{rot} + \frac{\hbar}{2\mu}(k - k_{rot})^2. \tag{25}$$

Then expansion of the two corresponding roots, which we will denote as $k_{2\,z}$ and $k_{3\,z}$, in terms of the small parameter $(\Delta_{rot} - \omega)$, contains summands $\sim \sqrt{\Delta_{rot} - \omega}$, which determine the asymptotic behavior of $\omega(k_\tau)$ in the vicinity of the roton gap:

$$k_c - k_\tau = d\cdot\sqrt{\Delta_{rot} - \omega}. \tag{26}$$

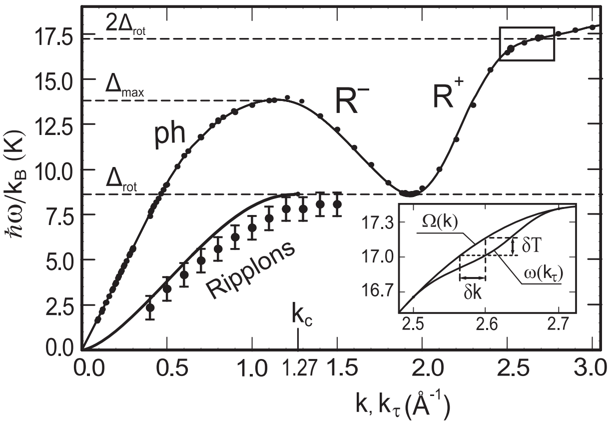

Fig. 1. *Bulk and surface excitations of He II. Dots are experimental data on neutron scattering [1, 5]; solid lines are numerical approximation to the bulk spectrum and the derived dispersion for the ripplons. Inset shows the new ripplon branch, with* $\delta T \approx 1.6\cdot 10^{-3}K$, $\delta k \approx 0.7\cdot 10^{-3}\text{Å}^{-1}$.

Thus the ripplon dispersion curve (Fig. 1) ends at a point (k_c, Δ_{rot}) with zero derivative, where $k_c < k_{rot}$ is the root of $\omega(k_c) = \Delta_{rot}$, which should be evaluated numerically (see below). At higher frequencies energy-momentum conservation laws allow ripplons' decay into rotons, and they are unstable. The constant d can be expressed through the parameters of the spectrum at Δ_{rot}:

$$d = \Big|\frac{b}{a+c}\Big|;\quad a = \frac{2\sigma}{\rho_0}\frac{\Delta_{rot}^2}{k_c^3};\quad b = 2k_{rot}\sqrt{\frac{2\mu/\hbar}{k_{rot}^2 - k_c^2}};$$

$$c = ik_c\Big\{k_{1\,z}^{-1}(\Delta_{rot},k_c) + \sum_{i\geq 4}k_{i\,z}^{-1}(\Delta_{rot},k_c) - \sum_{i\geq 2}k_{i\,z}^{-1}(0,k_c)\Big\}.$$

Numerical solution gives us the dispersion curve $\omega(k_\tau)$ for all frequencies, and gives $k_c = 1.27\,\text{Å}^{-1}$. It also reveals an unexpected new branch of ripplons at very high frequencies, close to $2\Delta_{rot}$. This branch is glued to the bulk dispersion curve from below, and is stable with regard to decay into bulk excitations.

5. PHONONS AND ROTONS AT INTERFACE WITH A SOLID

Now let us consider the problem of quasiparticles of the superfluid interacting with the interface with another medium. In this case the asymptote of the solution at $z \to \infty$ should correspond to isolated traveling quasiparticles, and at least one root $k_{i\,z}$ should be real. The easiest way to make sense of the general solution (13) in this case is the usual roots shifting from the real line into the upper or lower complex half-planes. In our case, however, there is additional condition imposed: the number of real roots shifted down must be equal to the number of roots shifted up, so that the index of density I remains equal to zero. This was an essential ingredient for validity of the general solution[1].

[1] The index of I in our case is essentially the difference between the number of its zeros in $\mathcal{C}_+$ and $\mathcal{C}_-$. For mathematical details we must refer the reader to [9].

The real roots $k_{i\,z}$ go in pairs, and for superfluid helium their maximum number is $N = 6$. We will consider this most general case, as the others $N = 2, 4$ ($N = 0$ was discussed in the previous section) can be deduced in the same way, and the final results can be directly obtained from the general formulas.

A solution is obtained from (13) by picking arbitrary three out of the set of six real roots, which are then shifted in the upper half-plane and after inverse Fourier transform give traveling waves in the coordinate space. The overall number of linear-independent solutions is thus $C_6^3 = 20$. Physically relevant for the problem of waves/quasiparticles' interaction with an interface are four of the set, which contain no more than one particle traveling towards the interface, i.e. in the negative direction of axis z, at $z \to \infty$. The basic, and most interesting, solution, is the one that contains all three quasiparticles traveling away from the interface. We will denote it as P_{out}.

We denote by $k_{1,2,3}$ the three positive roots of equation $\Omega^2(k) = \omega^2$ in the order of their absolute values, such that 1 stands for phonons, 2 for R^- rotons, and 3 for R^+ rotons. R^- rotons have negative group velocity, and thus propagate in the opposite direction of their wave-vectors. Therefore the P_{out} solution will contain traveling waves corresponding to $k_{1,2,3}$, with the signs defined as

$$0 < k_{1\,z} < (-k_{2\,z}) < k_{3\,z}. \tag{27}$$

We denote the remaining, initially complex roots of Eq. (12) in $\mathcal{C}_+$, by subscripts $i > 3$.

Solid's phonon incident. The P_{out} solution is the one that is realized in the fluid when a quasiparticle is incident on the interface from the solid. Its amplitude is derived by using boundary conditions discussed below, which gives us the total energy transmission coefficient D and the partial ones D_i, which describe the portions of the energy flow of the incident wave transferred into each of the created ones in the fluid. But it is important, that the *relative* amplitudes and energy flows of the asymptotically distinct three waves are determined by the structure of solution P_{out} alone, with no regard to parameters of the interface. Calculating the amplitudes of each wave as corresponding residues r_i (16), (18) and group velocities, we derive the relative energy transmission coefficients:

$$\frac{D_i}{D} = \frac{k_{i\,z}}{k_{i\,z}+k_{j\,z}+k_{k\,z}} \cdot \frac{k_{i\,z}+k_{j\,z}}{k_{i\,z}-k_{j\,z}} \frac{k_{i\,z}+k_{k\,z}}{k_{i\,z}-k_{k\,z}}, \tag{28}$$

where the subscripts $\{i, j, k\}$ form a permutation of $\{1, 2, 3\}$. These expressions do not depend on $k_{i\,z}$ with $i > 3$. It can be verified directly that $\sum D_i = D$. Due to (27), $D_2 \ll D_{1,2}$ (Fig. 2), so R^- rotons are weakly created by a solid's phonon.

When we consider the full problem of a solid's phonon incident on the interface with superfluid helium, we use boundary conditions

$$P\big|_{z=-0} = P\big|_{z=+0}; \quad \mathrm{v}_z\big|_{z=-0} = \mathrm{v}_z\big|_{z=+0}. \tag{29}$$

They imply that ω and $\mathbf{k}_\tau$ are the same for all harmonic summands on both sides of the interface. For traveling waves this fixes all angles of transmission and reflection, measured from the normal, via a generalization of Snell's law

$$\frac{\sin\theta_i}{s_i(\omega)} = \frac{\sin\theta_j}{s_j(\omega)} \quad \forall i, j = 0, 1, \dots \tag{30}$$

where $s_i(\omega) = \omega/k_i(\omega)$ are phase velocities; subscript 0 denotes solid's phonons.

In the scalar model the solid's phonon corresponds to a longitudinal wave; its dispersion is almost linear in the frequency scale of helium dispersion curve. Then the solution in the solid is just a sum of incident and reflected waves, and in the fluid it is P_{out}. Then using the b.c. (29), we express the amplitudes of all the waves through the amplitude of the incident one, and calculating energy flows in each of them, derive full energy transmission coefficient

$$D(\omega, k_\tau) = \frac{4\,\mathrm{Re}\,k_s k_q}{|k_s + k_q|^2}, \quad \text{where} \tag{31}$$

$$k_q = k_{1\,z} + \sum_{i=2} \big[k_{i\,z} - k_{i\,z}(\omega=0)\big]; \; k_s = \frac{\rho_q}{\rho_0}\, k_{0\,z}, \tag{32}$$

and ρ_0 is equilibrium density of the solid. The partial transmission coefficients are then given by (28).

Due to smallness of parameters ρ_q/ρ_0 and s_i/s_0 for helium and common solids, the transmission coefficients D and D_i for real interfaces with helium are obtained in the effective limit $\rho_q \to 0$. In this case the transmission angles θ_i are small due to (30) and

$$D_i(\omega, \theta_i) \approx D_i(\omega, 0)\cos\theta_i. \tag{33}$$

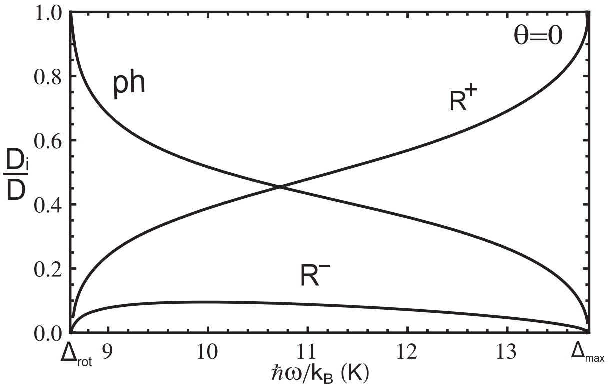

Fig. 2. *Relative transmission coefficients $D_i(\omega)/D(\omega)$ at normal incidence. They do not depend on the parameters of the interface, and are only determined by dispersion curve $\Omega(k)$*

A helium quasiparticle is incident. In this case the asymptote of the solution in $z > 0$ must contain one traveling wave traveling towards the interface. If the incident quasiparticle carries subscript i, then the solution can be presented as a linear combination of P_{out} and a solution, in which $k_{i\,z}$ in the denominator is formally replaced by $(-k_{i\,z})$ (numerator stays the same):

$$P_{in}^{(i)} = P_{out}\Big|_{k_{i\,z} \mapsto -k_{i\,z}}. \tag{34}$$

Using the same scheme as used above, we derive the transmission and reflection coefficients. The transmission coefficients D_i', when expressed through conserved quantities ω and k_τ, obey the principle of detailed balance $D_i'(\omega,k_\tau)=D_i(\omega,k_\tau)$.

There are nine reflection coefficients R_{ij}, where the first subscript denotes the type of incident quasiparticle, and the second – of the reflected one:

$$R_{ii}=\left|\frac{(k_{i\,z}+k_{j\,z})(k_{i\,z}+k_{k\,z})}{(k_{i\,z}-k_{j\,z})(k_{i\,z}-k_{k\,z})}\right|^2\cdot\left|1-\frac{2k_{i\,z}}{k_s+k_q}\right|^2; \tag{35}$$

$$R_{ij}=\frac{|4k_{i\,z}k_{j\,z}|}{(k_{i\,z}-k_{j\,z})^2}\left|\frac{(k_{i\,z}+k_{k\,z})(k_{j\,z}+k_{k\,z})}{(k_{i\,z}-k_{k\,z})(k_{j\,z}-k_{k\,z})}\right| \times\left|1-\frac{k_{i\,z}+k_{j\,z}}{k_s+k_q}\right|^2 \quad \text{for} \quad i\neq j. \tag{36}$$

The formulas for R_{ij} in the form (35,36) hold for the case when some of the roots $k_{i\,z}$ are complex. For example, if $k_{1\,z}$ and $k_{2\,z}$ are complex, then R_{11} is given by (35), where the first factor is reduced to unity. It is then trivial to show that $R_{11}+D_1=1$. It can be also directly verified in each case, that energy is always conserved $\sum_j R_{ij}+D_i=1$.

6. CONCLUSIONS

We used the description of superfluid helium as a continuous medium at interatomic scales, obeying equations of nonlocal hydrodynamics. Their solutions apply both to description of surface excitations, ripplons, and interaction of the bulk excitations, phonons and rotons, with interfaces. Ripplons' dispersion relation is derived, and conforms well with experimental data. It is shown to end in a point with zero derivative at the frequency of the roton gap; new ripplon branch is found at very high frequencies. Creation probabilities of all quasiparticles are derived when any quasiparticle at the interface between superfluid helium and a solid. It is shown that R^- rotons weakly interact with interfaces, which also explains experiments.

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ФОНОНЫ, РОТОНЫ И РИПЛОНЫ НА ГРАНИЦАХ РАЗДЕЛА

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Для описания тепловых возбуждений сверхтекучего гелия 4 используется нелокальная гидродинамика, в которой дисперсионное соотношение является входным параметром. Для решения нелокальных уравнений жидкости в полупространстве используется метод Винера-Хопфа. Вычислено дисперсионное соотношение поверхностных мод гелия, риплонов; численное решение выявляет существование новой ветви. Это же решение описывает взаимодействие фононов и ротонов с границей раздела с твердым телом. Найдены все вероятности рождения квазичастиц и объяснена слабость взаимодействия ротонов с отрицательной дисперсией с границей раздела.

ФОНОНИ, РОТОНИ І РІПЛОНИ НА ГРАНИЦЯХ РОЗПОДІЛУ

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Для опису теплових збуджень надплинного гелію 4 використовується нелокальна гідродинаміка, в якій дисперсійне співвідношення є вхідним параметром. Для розв'язку нелокальних рівнянь рідини в півпросторі використовується метод Вінера-Хопфа. Обчислено дисперсійне співвідношення поверхневих мод гелію, ріплонів; чисельний розв'язок виявляє їх нову гілку. Те ж рішення описує взаємодію фононів і ротонів із границею розподілу із твердих тілом. Знайдені всі вірогідності народження квазічастинок та пояснена слабкість взаємодії ротонів з від'ємною дисперсією із границею розподілу.