DOMAIN OF APPLICABILITY OF VARIOUS APPROACHES TO DESCRIPTION OF ELECTROMAGNETIC FIELD IN MEDIUM

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The problem of parameters, which are necessary for nonequilibrium electromagnetic field description, is investigated on the basis of the Bogolyubov reduced description method establishing the necessity of binary correlations in the minimal set of parameters taken into account in evolution equations. The corresponding theory can be built in terms of one-particle density matrices, Wigner distribution functions, simultaneous correlation functions of field operators, and generating functionals. The obtained results can be analyzed with the help of approaches elaborated in quantum optics. Various methods in theoretical and experimental research into field correlations are compared.

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1. INTRODUCTION

A statistical operator gives the most general description of field states, but from the point of view of experiments only reduced description of electromagnetic fields is possible. In all the cases it is necessary to choose physical quantities providing an adequate picture of nonequilibrium processes after transfer to averages. In our previous investigations several approaches to the construction of kinetic equations have been outlined. The used methods can be connected due to relatively simple relations expressing their key quantities through one another. At the same time quantum optics introduces non-simultaneous correlation functions and requires advanced methods.

2. REDUCED DESCRIPTION AND NECESSARY SET OF FIELD PARAMETERS

The Bogolyubov reduced description method (see, for example, [1]) can be a basis for the general consideration of the problem. Its starting point in this approach is the quantum Liouville equation for the statistical operator $\rho(t)$ of a system including electromagnetic field (subsystem f) and a medium (subsystem m)

$$\partial_t \rho(t) = -\frac{i}{\hbar}[\hat{H}, \rho(t)], \quad (\hat{H} = \hat{H}_\mathrm{f} + \hat{H}_\mathrm{m} + \hat{H}_\mathrm{mf}). \quad (1)$$

The method is based on the functional hypothesis describing a structure of the operator $\rho(t)$ at long times

$$\rho(t) \xrightarrow[t \gg \tau_0]{} \rho(\xi(t,\rho_0), \eta(t,\rho_0)) \quad (\rho_0 \equiv \rho(t=0)) \quad (2)$$

where reduced description parameters of field $\xi_\mu(t,\rho_0)$ and matter $\eta_a(t,\rho_0)$ are used. The set of parameters $\xi_\mu(t,\rho_0)$ is determined by the possibilities and traditions of experiments as well as by theoretical considerations. The development of the problem investigations has resulted in finding the main approximation for the statistical operator $\rho(\xi,\eta)$, so called a quasi-equilibrium statistical operator $\rho_q(Z(\xi), X(\eta))$(though it describes states which are far from the equilibrium) defined by the relations

$$\rho_q(Z,X) = \rho_\mathrm{f}(Z)\rho_m(X);$$

$$\rho_\mathrm{f}(Z) = \exp\{\Phi(Z) - \sum_\mu Z_\mu \hat{\xi}_\mu\},$$

$$\mathrm{Sp}_\mathrm{f}\rho_\mathrm{f}(Z) = 1, \quad \mathrm{Sp}_\mathrm{f}\rho_\mathrm{f}(Z(\xi))\hat{\xi}_\mu = \xi_\mu;$$

$$\rho_m(X) = \exp\{\Omega(X) - \sum_a X_a \hat{\eta}_a\},$$

$$\mathrm{Sp}_m\rho_m(X) = 1, \quad \mathrm{Sp}_m\rho_m(X(\eta))\hat{\eta}_a = \eta_a. \quad (3)$$

Practically all the electrodynamics of continuous media operates with average values of electromagnetic field $\xi_{\alpha n}(x,t)$: $\xi_{1n}(x,t) = E_n(x,t)$, $\xi_{2n}(x,t) = B_n(x,t)$. In this case operators $\hat{\xi}_\mu$ in (3) are $\hat{\xi}_{\alpha n}(x)$: $\hat{\xi}_{1n}(x) = \hat{E}_n(x)$, $\hat{\xi}_{2n}(x) = \hat{B}_n(x)$ which in the Coulomb gauge are given by expressions

$$\hat{E}_n(x) = i\sum_{\alpha k}\frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}} e_{\alpha k n}(c_{\alpha k} - c^+_{\alpha,-k})e^{ikx},$$

$$\hat{B}_n(x) = \varepsilon_{nlm}\times$$

$$\times i\sum_{\alpha k}\frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}}\tilde{k}_l e_{\alpha k m}(c_{\alpha k} + c^+_{\alpha,-k})e^{ikx} \quad (4)$$

(we use standard notations of quantum electrodynamics; $\tilde{k}_l \equiv k_l/k$). Nevertheless in paper [2] it has

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been pointed out that for this choice of operators $\hat{\xi}_\mu$ the statistical operator $\rho_{\rm f}(Z)$ does not exist because of containing a linear form on Bose field operators $c_{\alpha k}$, $c^+_{\alpha k}$ in the exponent. The situation can be corrected with the availability of a quadratic form of the operators in the exponent; hence the statistical operator has a structure

$$\rho_{\rm f}(Z) = \exp\{\Phi(Z) - \sum_{\alpha k, \alpha' k'} Z^{\alpha\alpha'}_{kk'} c^+_{\alpha k} c_{\alpha' k'} - \\ - (\sum_{\alpha k, \alpha' k'} \tilde{Z}^{\alpha\alpha'}_{kk'} c^+_{\alpha k} c^+_{\alpha' k'} + \sum_{\alpha k} Z^{\alpha}_{k} c^+_{\alpha k} + h.c.)\}. \quad (5)$$

In fact, it means that the minimal set of reduced description parameters of electromagnetic field contains its binary fluctuations (correlations) besides its average strength though the average field can be absent.

3. VARIOUS APPROACHES TO CONSTRUCTING THE KINETIC EQUATIONS WITH BINARY CORRELATIONS ACCOUNT

In order to describe binary fluctuations of the field, normal $n^{\alpha\alpha'}_{kk'}(t) = {\rm Sp}\rho(t) c^+_{\alpha k} c_{\alpha' k'}$ and anomalous $\tilde{n}^{\alpha\alpha'}_{kk'}(t) = {\rm Sp}\rho(t) c_{\alpha k} c_{\alpha' k'}$ one-particle density matrices (DM) can be used (a T-1 theory) as well as connected with them normal

$$\mathrm{f}^{\alpha\alpha'}_{k}(x,t) = {\rm Sp}\rho(t) \hat{\mathrm{f}}^{\alpha\alpha'}_{k}(x)$$

and anomalous

$$\tilde{\mathrm{f}}^{\alpha\alpha'}_{k}(x,t) = {\rm Sp}\rho(t) \hat{\tilde{\mathrm{f}}}^{\alpha\alpha'}_{k}(x)$$

Wigner distribution functions (WDF) (a T-2 theory) where

$$\hat{\mathrm{f}}^{\alpha\alpha'}_{k}(x) = \sum_q c^+_{\alpha, k-q/2} c_{\alpha', k+q/2} e^{iqx},$$

$$\hat{\tilde{\mathrm{f}}}^{\alpha\alpha'}_{k}(x) = \sum_q c_{\alpha, k+q/2} c_{\alpha', -k+q/2} e^{iqx}. \quad (6)$$

The anomalous DM and WDF are absent in a state with the statistical operator $\rho_{\rm f}(Z)$ at $\tilde{Z}^{\alpha\alpha'}_{kk'} = 0$ and average electromagnetic field equals zero at $Z^{\alpha}_{k} = 0$. The terminology "normal-anomalous" is connected with the concept of spontaneous break of symmetry; herewith from this point of view a nonzero average field value is also a consequence of some symmetry breakdown.

Electromagnetic field fluctuations can be described also with average values of field operators

$$\langle \xi^x_{\alpha n} \xi^{x'}_{\alpha' l} \rangle_t = \frac{1}{2} {\rm Sp}\rho(t) \{\hat{\xi}_{\alpha n}(x), \hat{\xi}_{\alpha' l}(x')\}, \quad (7)$$

or corresponding correlation functions

$$(\xi^x_{\alpha n} \xi^{x'}_{\alpha' l})_t = \langle \xi^x_{\alpha n} \xi^{x'}_{\alpha' l} \rangle_t - \xi_{\alpha n}(x,t) \xi_{\alpha' l}(x',t)$$

(a T-3 theory).

The unique relationship of such theories proceeds from the formulas (4) and their consequence

$$c_{\alpha k} = \frac{e^*_{\alpha k n}}{(8\pi\omega_k \hbar V)^{1/2}} \int d^3x \left[\frac{\hat{Z}_n(x)}{k} - i\hat{E}_n(x) \right] e^{-ikx}. \quad (8)$$

($\hat{Z}_n(x) \equiv {\rm rot}_n \hat{B}(x)$). T-2 and T-3 theories allow describing a spatial behavior of the electromagnetic field in medium. Temporal equations for one-particle DM and WDF are called kinetic equations for photons in medium.

In the paper [3] a theory of T-1 type has been built for electromagnetic field in equilibrium plasma medium. The corresponding kinetic equations have a form

$$\partial_t g^{\alpha\alpha'}_{kk'} = i(\Omega_k - \Omega_{k'}) g^{\alpha\alpha'}_{kk'} - \\ - (\nu_k + \nu_{k'})(g^{\alpha\alpha'}_{kk'} - n_k \delta_{\alpha\alpha'} \delta_{kk'}),$$
$$\partial_t x_{\alpha k} = -(i\Omega_k + \nu_k) x_{\alpha k} + (\nu_k + i\omega_k \chi_k) x^*_{\alpha, -k} \quad (9)$$

where

$$g^{\alpha\alpha'}_{kk'}(t) = n^{\alpha\alpha'}_{kk'}(t) - x^*_{\alpha k}(t) x_{\alpha' k'}(t),$$
$$x_{\alpha k}(t) = {\rm Sp}\rho(t) c_{\alpha k},$$
$$\Omega_k = \omega_k(1 - 2\pi\chi_k), \quad \nu_k = 2\pi\sigma_k \quad (10)$$

(Ω_k is a photon spectrum in the medium, n_k is Planck distribution with a medium temperature). The second equation (9) is actually a Maxwell equation with account of a material equation, at that σ_k and χ_k are conductivity and magnetic susceptibility of the medium expressed via the Green function of currents. In terms of WDF in the case of weakly nonuniform state of the system such kinetic equation takes the form

$$\partial_t \mathrm{f}^{\alpha\alpha'}_{k} = -\frac{\partial\Omega_k}{\partial k_n} \frac{\partial \mathrm{f}^{\alpha\alpha'}_{k}}{\partial x_n} + \frac{1}{4} \frac{\partial^2 \nu_k}{\partial k_n \partial k_l} \frac{\partial^2 \mathrm{f}^{\alpha\alpha'}_{k}}{\partial x_n \partial x_l} - \\ -2\nu_k(\mathrm{f}^{\alpha\alpha'}_{k} - n_k \delta_{kk'} \delta_{\alpha\alpha'}). \quad (11)$$

4. KINETICS OF ELECTROMAGNETIC FIELD INTERACTING WITH NONEQUILIBRIUM SYSTEM OF EMITTERS

In our papers (see [4]) electrodynamics in a medium consisting of two-level emitters has been built. Such a theory emerges in the course of research into the Dicke superfluorescence [5] on the basis of the Bogolyubov reduced description method. A standard approach lies in the framework of a theory of T-3 type.

Emitter subsystem is regarded as nonuniform and it is convenient to describe it with a density of emitter energy

$$\hat{\eta}_a: \quad \hat{\varepsilon}(x) = \hbar\omega \sum_{1 \le a \le N} \hat{r}_{az} \delta(x - x_a). \quad (12)$$

Operators of the reduced description parameters of the field subsystem in the developed theory are

$$\hat{\xi}_\mu: \quad \hat{\xi}_{\alpha n}(x), \quad \frac{1}{2}\{\hat{\xi}_{\alpha n}(x), \hat{\xi}_{\alpha' l}(x')\}.$$

Maxwell equations in terms of averages have the form

$$\partial_t E_n(x,t) = c\,\mathrm{rot}_n B(x,t) - 4\pi J_n(x,\varepsilon(t),\xi(t)),$$
$$\partial_t B_n(x,t) = -c\,\mathrm{rot}_n E(x,t) \quad (13)$$

with a material equation

$$J_n(x,\varepsilon,\xi) = \int dx'\sigma(x-x',\varepsilon(x))E_n(x')+$$
$$+c\int dx'\chi(x-x',\varepsilon(x))Z_n(x') \quad (14)$$

where Fourier transforms of functions $\sigma(x,\varepsilon)$, $\chi(x,\varepsilon)$ are conductivity $\sigma_k(\varepsilon)$ and magnetic susceptibility $\chi_k(\varepsilon)$ of the system. Equations for field correlations acquire the form

$$\partial_t(E_n^x E_l^{x'}) = c\,\mathrm{rot}_n(B^x E_l^{x'}) + c\,\mathrm{rot}'_l(E_n^x B^{x'})-$$
$$-4\pi(J_n^x E_l^{x'}) - 4\pi(E_n^x J_l^{x'}),$$
$$\partial_t(E_n^x B_l^{x'}) = c\,\mathrm{rot}_n(B^x B_l^{x'}) - c\,\mathrm{rot}'_l(E_n^x E^{x'})-$$
$$-4\pi(J_n^x B_l^{x'}),$$
$$\partial_t(B_n^x E_l^{x'}) = -c\,\mathrm{rot}_n(E^x E_l^{x'}) + c\,\mathrm{rot}'_l(B_n^x B^{x'})-$$
$$-4\pi(B_n^x J_l^{x'}),$$
$$\partial_t(B_n^x B_l^{x'}) = -c\,\mathrm{rot}_n(E^x B_l^{x'}) - c\,\mathrm{rot}'_l(B_n^x E^{x'}). \quad (15)$$

Current-field correlations are expressed via field correlations by material equations obeying the Onsager principle.

5. ENERGY FLUXES IN MEDIUM AND CORRELATION FUNCTIONS

In kinetics of electromagnetic field the energy fluxes in medium is a problem of interest. An operator of energy flux is given by the formula

$$\hat{q}_n(x) = \frac{c}{8\pi}\varepsilon_{nlm}\{\hat{E}_l(x),\hat{B}_m(x)\}. \quad (16)$$

According to the reduced description method, in the theory taking into account only binary field correlations exact averages of binary field functions are determined by the quasi-equilibrium field distribution. Therefore the energy flux is expressed exactly via one-particle DM or WDF. For example, if the average field is absent, we come to an exact formula

$$q_n(x) = \hbar c^2 \sum_{\alpha\alpha'}\int \frac{d^3k}{(2\pi)^3}\varphi_n^{\alpha\alpha'}(k,-i\frac{\partial}{\partial x})\mathrm{f}_k^{\alpha\alpha'}(x) \quad (17)$$

where the notation is used

$$\varphi_n^{\alpha_1\alpha_2}(k,q) = \varphi_n^{\alpha_1\alpha_2}(k-q/2,k+q/2),$$
$$\varphi_n^{\alpha_1\alpha_2}(k_1,k_2) \equiv \frac{1}{2}(\delta_{nl}\delta_{ms}-\delta_{ml}\delta_{ns})(k_1k_2)^{1/2}\times$$
$$\times\{\tilde{k}_{1l}e^*_{\alpha_1k_1s}e_{\alpha_2k_2m} + \tilde{k}_{2l}e^*_{\alpha_1k_1m}e_{\alpha_2k_2s}\}. \quad (18)$$

In a weakly nonuniform state this formula leads to a well-known elementary result

$$q_n(x) = \sum_\alpha \int \frac{d^3k}{(2\pi)^3}\omega_k\hbar\frac{\partial\omega_k}{\partial k_n}\mathrm{f}_k^{\alpha\alpha}(x). \quad (19)$$

Formula (17) should be put in the basis of the theory of radiation transfer. The simplest consideration is based on approximate expression (19). Radiation transfer can be described by specific intensity of radiation

$$I_\omega^{\alpha\alpha'}(n,x) \equiv \frac{\omega^3\hbar}{(2\pi)^3c^2}\,\mathrm{f}_k^{\alpha\alpha'}(x)\Big|_{k=n\frac{\omega}{c}} \quad (20)$$

($|n|=1$). This definition gives formulas for energy flux of the electromagnetic field

$$q_l(x) = \sum_\alpha \int I_\omega^{\alpha\alpha}(n,x)n_l d\omega d\Omega_n \quad (21)$$

and its energy density

$$\varepsilon(x) = \frac{1}{c}\sum_\alpha \int I_\omega^{\alpha\alpha}(n,x)d\omega d\Omega_n \quad (22)$$

which are in common use in the radiation transfer theory [6].

Equation of the radiation transfer follows from definition (20) and kinetic equation (11)

$$\frac{\partial I_\omega^{\alpha\alpha'}(n,x)}{\partial t} = -c_\omega n_l\frac{\partial I_\omega^{\alpha\alpha'}(n,x)}{\partial x_l}-$$
$$-2\nu_\omega\{I_\omega^{\alpha\alpha'}(n,x) - I_\omega\delta_{\alpha\alpha'}\}+$$
$$+\{a_\omega n_l n_m + b_\omega\delta_{lm}\}\frac{\partial^2 I_\omega^{\alpha\alpha'}(n,x)}{\partial x_l\partial x_m} \quad (23)$$

where the notations

$$\frac{\partial\Omega_k}{\partial k_l}\Big|_{k=\frac{\omega}{c}n} \equiv c_\omega n_l, \quad \nu_k|_{k=\frac{\omega}{c}n} \equiv \nu_\omega,$$
$$\frac{\partial^2\nu_k}{\partial k_l\partial k_m}\Big|_{k=\frac{\omega}{c}n} \equiv 4\{a_\omega n_l n_m + b_\omega\delta_{lm}\},$$
$$I_\omega \equiv \frac{\omega^3\hbar}{(2\pi)^3c^2}\frac{1}{e^{\hbar\omega/T}-1} \quad (24)$$

are introduced. Usually this equation is written for stationary states and without correction given with the last term.

6. FIELD CORRELATION PROPERTIES IN QUANTUM OPTICS

The most general approach of quantum optics to the statistical properties of light is based on the technique of photon counting and the concept of an ideal quantum detector. Its operation analysis by Glauber [7] has led to the conclusion that the simplest observable correlation quantity describing electromagnetic field is

$$\mathrm{Sp}\{\rho\hat{E}_n^{(-)}(x,t)\hat{E}_l^{(+)}(x,t)\}. \quad (25)$$

It can be measured in the experiments of the first order (for example, in the Young scheme). Here ρ is a statistical operator of the field; $\hat{E}_n^{(+)}(x,t)$, $\hat{E}_n^{(-)}(x,t)$

are positive-frequency and negative-frequency parts of the electric field operator (4) in the interaction picture:

$$\hat{E}_n(x,t) = \hat{E}_n^{(+)}(x,t) + \hat{E}_n^{(-)}(x,t),$$

$$\hat{E}_n^{(-)}(x,t) = \hat{E}_n^{(+)}(x,t)^+, \qquad \hat{E}_n^{(+)}(x,t) =$$

$$= i\sum_{k\alpha} \frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}} e_{\alpha kn} c_{\alpha k}\, e^{i(kx-\omega_k t)}. \qquad (26)$$

In experiments of the second order (for example, the Hunbury Brown-Twiss scheme based on photon detection coincidence) observable correlation quantity describing electromagnetic field is

$$\mathrm{Sp}\{\rho \hat{E}_{n_1}^{(-)}(x_1,t_1)\hat{E}_{n_2}^{(-)}(x_2,t_2)\times$$

$$\times \hat{E}_{l_2}^{(+)}(x_2,t_2)\hat{E}_{l_1}^{(+)}(x_1,t_1)\}. \qquad (27)$$

In more complicated experiments all correlation functions of the form

$$G^{ss'}_{n_1\ldots n_s,l_1\ldots l_{s'}}(y_1\ldots y_s, y'_1\ldots y'_{s'}) =$$

$$= \mathrm{Sp}\{\rho \hat{E}_{n_1}^{(-)}(y_1)\ldots\hat{E}_{n_s}^{(-)}(y_s)\hat{E}_{l_1}^{(+)}(y'_1)\ldots\hat{E}_{l_{s'}}^{(+)}(y'_{s'})\} \qquad (28)$$

can be considered as observables quantities in quantum optics ($y \equiv (x,t)$) [8]. Statistical operators built with the Bogolyubov reduced description method can be used in this formula as a statistical operator.

Note that appearance of operators $\hat{E}_n^{(+)}(x,t)$, $\hat{E}_n^{(-)}(x,t)$ in formulas (25), (27), (28) instead of the total field operator $\hat{E}_n(x,t)$ is a consequence of high frequencies of electromagnetic field of visible spectrum.

Correlation functions of the first order G^{11} can be expressed through the one-particle density matrix $n^{\alpha\alpha'}_{kk'}$ exactly and those of the second order G^{22} can be expressed through it only approximately. The most interesting quantum correlation effects (such as photon antibunching, squeezing, sub-Poissonian statistics) are described with correlation functions concerning different time moments [8].

Introduced correlation functions contain averages over normal product of operators of electromagnetic field. In this case it is very convenient to use the Glauber formalism of coherent states. Statistical operator of electromagnetic field can be represented in the Glauber-Sudarshan form [7]

$$\rho = \int d^2z P(z,z^*)|z\rangle\langle z|, \qquad (29)$$

where $\{|z\rangle\}$ is basis of the coherent state representation

$$c_{\alpha k}|z\rangle = z_{\alpha k}|z\rangle \quad (z \equiv \{z_{\alpha k}\}),$$

$$\frac{1}{\pi}\int d^2z|z\rangle\langle z| = 1, \qquad d^2z = \prod_{\alpha k} d^2 z_{\alpha k}$$

$$z_{\alpha k} \equiv z'_{\alpha k} + iz''_{\alpha k}, \quad d^2 z_{\alpha k} \equiv dz'_{\alpha k}dz''_{\alpha k}. \qquad (30)$$

In this formalism correlation functions take the form

$$G^{ss'}_{n_1\ldots n_s,l_1\ldots l_{s'}}(y_1\ldots y_s, y'_1\ldots y'_{s'}) =$$

$$= \int d^2zP(z)E^*_{n_1 z}(y_1)\ldots E^*_{n_s z}(y_s)E_{l_1 z}(y'_1)\ldots E_{l_{s'} z}(y'_{s'}) \qquad (31)$$

where

$$E_{nz}(x,t) \equiv i\sum_{k\alpha} \frac{(2\pi\hbar\omega_k)^{1/2}}{V^{1/2}} e_{\alpha kn} z_{\alpha k}\, e^{i(kx-\omega_k t)}. \qquad (32)$$

7. FORMALISM OF GENERATING FUNCTIONAL

A generating functional can be introduced for a statistical operator

$$F(u,u^*) = \mathrm{Sp}\rho e^{\sum\limits_{\alpha k} u_{\alpha k}c^+_{\alpha k}} e^{-\sum\limits_{\alpha k} u^*_{\alpha k}c_{\alpha k}}, \qquad (33)$$

the functional giving possibility to calculate averages

$$\mathrm{Sp}\rho c^+_{\alpha_1 k_1}\ldots c^+_{\alpha_s k_s} c_{\alpha'_1 k'_1}\ldots c_{\alpha'_{s'} k'_{s'}} =$$

$$= (-1)^{s'} \left.\frac{\partial^{s+s'}F(u,u^*)}{\partial u_{\alpha_1 k_1}\ldots\partial u_{\alpha_s k_s}\partial u^*_{\alpha'_1 k'_1}\ldots\partial u^*_{\alpha'_{s'} k'_{s'}}}\right|_{u,u^*=0} \qquad (34)$$

and the Glauber-Sudarshan distribution

$$P(z,z^*) = \frac{1}{\pi}\int d^2u\, F(u,u^*) e^{\sum\limits_{\alpha k}\{u^*_{\alpha k}z_{\alpha k} - u_{\alpha k}z^*_{\alpha k}\}}. \qquad (35)$$

Formalism of generating functions was widely discussed in the frame of the reduced description method (see, for example, [1, 9]). In this theory (a T-4 theory) the quantum Liouville equation is written in the form of equation for the generating functional $F(z,z^*)$ and integral equations of the reduced description method are formulated as equations for this functional. In this theory generating functional for the quasi-equilibrium statistical operator $\rho_{\mathrm{f}}(Z(n))$ (see (5))

$$\rho_{\mathrm{f}}(Z) = \exp\{\Phi(Z) - \sum_{\alpha k,\, \alpha' k'} Z^{\alpha\alpha'}_{kk'} c^+_{\alpha k}c_{\alpha' k'}\},$$

$$\mathrm{Sp}\rho_{\mathrm{f}}(Z(n))c^+_{\alpha k}c_{\alpha' k'} = n^{\alpha\alpha'}_{kk'} \qquad (36)$$

is given by expression

$$F(u,u^*) = e^{-\sum\limits_{\alpha k,\,\alpha' k'} n^{\alpha\alpha'}_{kk'} u^*_{\alpha k}u_{\alpha' k'}}. \qquad (37)$$

An example of the T-4 theory is given by the paper [10] where kinetics of classical electromagnetic field in equilibrium medium was studied with taking into account all (not only binary) correlations of the field as reduced description parameters.

8. CONCLUSIONS

Kinetic theory of electromagnetic field in media has choosing a set of parameters describing nonequilibrium states of the field as a starting point with necessity. The minimal set of such parameters includes binary correlations of field amplitudes. The corresponding mathematical apparatus uses different structures of averages: one-particle density matrices, Wigner distribution functions, and conventional simultaneous correlation functions of field operators. All approaches can be connected with each other due to the possibility of expressing the main correlation parameters in various forms. The reduced description method elucidates the construction of kinetic equations in electrodynamics of continuous media (plasma, complex of two-level emitters) and radiation transfer theory. The obtained results can be analyzed with the help of approaches elaborated in quantum optics.

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References

1. A.I. Akhiezer, S.V. Peletminsky. *Methods of Statistical Physics.* Oxford: "Pergamon Press", 1981, 368 p.
2. S.V. Peletminskii, V.I. Prikhod'ko, V.S. Shchelokov. Behavior of electrodynamic Green's functions at low frequencies // *Theoretical and Mathematical Physics.* 1975, v. 25 (1), p. 986-991.
3. A.I. Sokolovsky, A.A. Stupka. Description of electromagnetic field in medium by intensities and one-photon density matrix // *The Journal of Kharkiv National University. Physical series: Nuclei, Particles, Fields.* 2004, v. 642, iss. 25 (3), p. 97-100 (in Ukrainian).
4. S.F. Lyagushyn, A.I. Sokolovsky. Kinetics of system of emitters and nonequilibrium electromagnetic field // *Physics of Particles and Nuclei.* 2010, v. 41 (7), p. 1035-1038.
5. N.N. Bogolyubov (Jr.), A.S. Shumovsky. *Superradiance.* Dubna: JINR, 1987, 88 p. (in Russian).
6. S. Chandrasekhar. *Radiative Transfer.* Dover Publications, 1960, 393 p.
7. R.J. Glauber. Optical coherence and photon statistics. In: *Quantum Optics and Electronics.* New York: "Gordon and Breach", 1964.
8. M.O. Scully, M.S. Zubairy. *Quantum Optics.* Cambridge University Press, 1997.
9. S.V. Peletminskii, A.A. Yatsenko. Generating functional method and virial expansions in nonequilibrium statistical mechanics // *Theoretical and Mathematical Physics.* 1970, v. 3 (2), p. 513-523.
10. A.I. Sokolovsky, A.A. Stupka. Classical fluctuation electrodynamics // *Condensed Matter Physics.* 2005, v. 8, N 4 (44), p. 685-700.

ОБЛАСТЬ ПРИМЕНИМОСТИ РАЗЛИЧНЫХ ПОДХОДОВ К ОПИСАНИЮ ЭЛЕКТРОМАГНИТНОГО ПОЛЯ В СРЕДЕ

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Проблема параметров, которые необходимы для описания неравновесного электромагнитного поля, исследуется на основе метода сокращённого описания Боголюбова, устанавливающего необходимость бинарных корреляций в минимальном наборе параметров, принимаемых во внимание в уравнениях эволюции. Соответствующая теория может быть построена в терминах одночастичных матриц плотности, вигнеровских функций распределения, одновременных корреляционных функций полевых операторов и производящих функционалов. Полученные результаты могут анализироваться с помощью подходов, разработанных в квантовой оптике. Сравниваются разные методы теоретического и экспериментального исследования корреляций поля.

ОБЛАСТЬ ЗАСТОСУВАННЯ РІЗНИХ ПІДХОДІВ ДО ОПИСУ ЕЛЕКТРОМАГНІТНОГО ПОЛЯ В СЕРЕДОВИЩІ

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Проблема параметрів, необхідних для опису нерівноважного електромагнітного поля, досліджується на основі методу скороченого опису Боголюбова, що встановлює необхідність бінарних кореляцій у мінімальному наборі параметрів, які беруться до уваги в рівняннях еволюції. Відповідна теорія може бути побудована мовою одночастинкових матриць густини, вігнерівських функцій розподілу, одночасних кореляційних функцій польових операторів і твірних функціоналів. Отримані результати можуть бути проаналізовані за допомогою підходів, розроблених у квантовій оптиці. Порівнюються різні методи теоретичного й експериментального дослідження кореляцій поля.