Section D. Theory of Irreversible Processes

ON THE LONG WAVE FLUCTUATIONS IN SYSTEMS OF PARTICLES INTERACTING WITH HYDRODYNAMIC MEDIA

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Our work offers a stochastic approach for description of long wave fluctuations in systems of particles interacting with hydrodynamic media. The presence of long-wave fluctuations in the system is caused by the random character of external force. The description of fluctuations evolution is based on the averaging of nonlinear dynamic equations over random external force acting on the system. The evolution equations for large scale fluctuations have been derived. The considered system can be a model of neutron transport in hydrodynamic media. We obtain the dynamic equations of long wave fluctuations generated by external random force in hydrodynamic media with multiplication and capture of neutrons and consider the influence of fluctuations over the stability of steady states of such systems.

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1. INTRODUCTION

The study of non-equilibrium long-wave fluctuations was started decades ago by works [1–3]. In these works it was shown that the presence of large scale fluctuations significantly affects process of relaxation toward equilibrium. For example, such fluctuations are responsible for the relaxations of some systems toward equilibrium according to power law (the "long hydrodynamic tails" theory). Since then numbers of works where devoted to study of role of non-equilibrium long-wave fluctuations in different relaxation processes. The work [4] offered the consistent microscopic approach for description of non-equilibrium long-wave fluctuations using reduced description method. The detailed study of generation of fluctuations by external random force was presented in the work [5]. This work continues our study of a system which consists of particles of one type which weakly interact with hydrodynamic medium, composed of particles of another type. This system can serve as a model for propagation of neutrons in hydrodynamic media. The kinetic and hydrodynamic evolution stages for such system where studied in our paper [6]. The role of non-equilibrium long wave fluctuations, generated by random initial conditions, in relaxation of such systems towards equilibrium was studied in paper [7] using the stochastic approach. In the present work we consider the non-equilibrium long-wave fluctuations generated by external random force and their role in the stability of the system.

2. AVERAGING OF DYNAMIC EQUATIONS OVER EXTERNAL RANDOM FORCE

We use the stochastic approach for derivation of evolution equations of non-equilibrium long wave fluctuations to obtain evolution equation for average values of description parameters and their correlations.

Let us consider a system which can be described by parameters $\zeta_a(\mathbf{x},t)$, and the motion equations for description parameters can be written in form:

$$\frac{\partial\hat{\zeta}_a(\mathbf{x},t)}{\partial t} = L_a\left(\mathbf{x},\hat{\zeta}(\mathbf{x}',t)\right) + \hat{Y}_a(\mathbf{x},t). \qquad (1)$$

Here $\hat{Y}_a(\mathbf{x},t)$ is the external random force, acting on our system. The symbol $\hat{}$ over Y_a and ζ_a denotes the random character of this values caused by randomness of the force.

The presence of external random force in equations (1) raises the question of averaging of these equations over the random forces $\hat{Y}_a(\mathbf{x},t)$. The averaging will result into general equations of fluctuative hydrodynamics for our system. Let us introduce the average values:

$$\zeta^s_{a_1,..,a_n}(\mathbf{x}_1,..,\mathbf{x}_s,t) = \langle\zeta_{a_1}(\mathbf{x}_1,t)\,..\,(\mathbf{x}_s,t)\rangle, \qquad (2)$$

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where the brackets $\langle ... \rangle$ mean averaging over the probability density of external random force $W\left[\hat{Y}\right]$:

$$\langle ... \rangle \equiv \int ...W\left[\hat{Y}\right] D\hat{Y}. \tag{3}$$

Next we introduce the generating functional of the average values $\zeta^{s}_{a_1,..,a_n}(\mathbf{x}_1,..,\mathbf{x}_s,t)$:

$$F(u,\zeta_a) = \left\langle \exp \int dx \hat{\zeta}_a(x,t)\, u_a(x) \right\rangle. \tag{4}$$

The average value of order s can be derived by differentiating this functional s times over $u_a(x)$:

$$\zeta^{s}_{a_1,..,a_n}(\mathbf{x}_1,..,\mathbf{x}_s,t) = \left. \frac{\delta^s F(u,\zeta_a)}{\delta u_{a_1..}(\mathbf{x}_1)\,..\delta u_{a_s..}(\mathbf{x}_s)} \right|_{u(\mathbf{x})=0}. \tag{5}$$

Finally we can define the generating functional $\mathcal{G}(u,\xi)$ of correlation functions $\xi_{a_1..a_s}(x_1,...,x_s;t)$:

$$\begin{gathered}\mathcal{G}(u,\xi) = \sum_{s=2}^{\infty} \frac{1}{s!} \int d\mathbf{x}_1 ... \int d\mathbf{x}_s \times \\ \times u_{a_1}(x_1) ... u_{a_s}(x_s)\, \xi^{(s)}_{a_1..a_s}(x_1,...,x_s;t),\end{gathered} \tag{6}$$

which is related to F by expressions

$$\begin{aligned} &F(u,\zeta_a) = \exp G(u,\zeta_a), \\ &G(u,\zeta) = G_0(u,\zeta) + \mathcal{G}(u,\xi), \\ &G_0(u,\zeta) \equiv \int d\mathbf{x} \zeta_a(\mathbf{x};t)\, u_a(\mathbf{x}), \end{aligned} \tag{7}$$

where $\zeta_a(\mathbf{x};t) \equiv \left\langle \hat{\zeta}_a(\mathbf{x};t) \right\rangle$. It is easy to show that any functional $A\left(\hat{\zeta}\right)$ averaged over external random force according (3) can be written as functional of correlation functions $A\left(\zeta, \xi^{(2)}, \xi^{(3)}, ...\right)$:

$$\left\langle A\left(\hat{\zeta}\right) \right\rangle = \exp\left(\mathcal{G}\left(\frac{\delta}{\delta \underline{\zeta}}, \xi\right)\right) A\left(\underline{\zeta}\right). \tag{8}$$

Now we can find the evolution equation for generating functional by differentiation over time of the definition (4):

$$\begin{gathered}\frac{\partial F(u,\zeta_a)}{\partial t} = \int d\mathbf{x} u_a(\mathbf{x}) \left\langle e^{\int u\hat{\zeta}} Y_a(\mathbf{x}) \right\rangle + \\ + \int d\mathbf{x} u_a(\mathbf{x}) \left\langle e^{\int u\hat{\zeta}} L_a\left(\mathbf{x}, \hat{\zeta}(\mathbf{x}',t)\right) \right\rangle.\end{gathered} \tag{9}$$

In order to shorten the latter expression the notation:

$$e^{\int u\hat{\zeta}} = \exp\left(\int d\mathbf{y} \zeta_a(\mathbf{y};t)\, u_a(\mathbf{y})\right)$$

was introduced. The first term in expression (9) represents the action of external random force on the system, the second one represents the evolution processes within the system. In order to get the evolution equation for generating functional we need to perform averaging in both terms. After simple but cumbrous transformations one can get final expression for the second term

$$\left\langle e^{\int u\hat{\zeta}} L_a\left(\mathbf{x}, \hat{\zeta}(\mathbf{x}',t)\right) \right\rangle = e^{G_0(u;\zeta)} e^{\mathcal{G}\left(u+\frac{\delta}{\delta\zeta};\xi\right)} L_a(\zeta), \tag{10}$$

though the averaging of the first term requires some estimations concerning the external force and some mathematical tricks. We introduce two generating functional: $P(v;Y_N(t))$ for average values

$$Y_{a_1..a_n}(\mathbf{x}_1..\mathbf{x}_n,t_1..t_n) \equiv \left\langle \hat{Y}_{a_1}(\mathbf{x}_1,t_1)\,..\hat{Y}_{a_1}(\mathbf{x}_1,t_1) \right\rangle$$

and $\mathcal{P}(v;b_N(t))$ for correlations of external random forces $b_{a_1..a_n}(\mathbf{x}_1,..,\mathbf{x}_n,t_1..t_n)$:

$$\begin{gathered}\mathcal{P}(v;b_n(t)) = \sum_{n=2}^{\infty} \frac{1}{n!} \int d\mathbf{x}_1 .. \int d\mathbf{x}_n \int dt_1 \int dt_n \times \\ \times v_{a_1}(\mathbf{x}_1,t_1)\,..v_{a_n}(\mathbf{x}_n,t_n)\, b_{a_1..a_n}(\mathbf{x}_1,..,\mathbf{x}_n,t_1..t_n),\end{gathered} \tag{11}$$

the same way as the functions $F(u,\zeta_a)$ and $\mathcal{G}(u,\xi)$ were introduced in (2)-(8). It is clear that averaging of any functional $B\left(\hat{Y}\right)$ over external random force $\hat{Y}$ can be given by relation:

$$\left\langle B\left(\hat{Y}\right) \right\rangle = \exp\left(\mathcal{P}\left(\frac{\delta}{\delta Y}, b\right)\right) B(Y), \tag{12}$$

similar to expression (7). Also the relation:

$$\begin{gathered}\left\langle \hat{Y}_a(\mathbf{x},t)\, Q\left(\hat{Y}\right) \right\rangle = Y_a(\mathbf{x},t)\, \mathrm{e}^{\mathcal{P}\left(\frac{\delta}{\delta Y},b\right)} Q(Y) + \\ + \left. \frac{\delta \mathcal{P}(v,b)}{\delta v_a(\mathbf{x},t)} \right|_{v_a \to \frac{\delta}{\delta Y_a}} \mathrm{e}^{\mathcal{P}\left(\frac{\delta}{\delta Y},b\right)} Q(Y)\end{gathered} \tag{13}$$

can be easily proved.

In order to perform further calculations we apply some restrictions on the external random force. Let us assume Gaussian distribution of external random force, existence of only pair correlations and possibility to separate spatial and time components of external force correlation function:

$$b_{ab}(\mathbf{x}_1,\mathbf{x}_2,t_1,t_2) = b_{ab}(\mathbf{x}_1,\mathbf{x}_2)\, g(t_1-t_2). \tag{14}$$

Here $b_{ab}(\mathbf{x}_1,\mathbf{x}_2)$ is the spatial part of correlation function, and the time part of correlation function $g(t_1-t_2)$ is supposed to be non zero only at small times $|t_1-t_2| < \tau_0$. When assuming $\tau_0 \to 0$ the time correlation function g can be handled as smeared delta function:

$$g(t) = g(-t), \quad \int\limits_{-\infty}^{\infty} g(t)\, dt = 1.$$

These assumptions allow us to perform averaging of the first term in (9):

$$\begin{gathered}\left\langle e^{\int u\hat{\zeta}} Y_a(\mathbf{x}) \right\rangle = Y_a(\mathbf{x},t)\, F(u,\zeta) + \\ + \frac{1}{2} \int d\mathbf{x}' u_b(\mathbf{x})\, b_{ab}(\mathbf{x},\mathbf{x}')\, F(u,\zeta),\end{gathered} \tag{15}$$

and after some transformations we come up to the general non-linear motion equation for the description parameters of the system and generating functional of correlation functions:

$$\frac{\partial \zeta_a(\mathbf{x},t)}{\partial t} = L_a\left(\mathbf{x},\hat{\zeta}(\mathbf{x}',t)\right) + Y_a(\mathbf{x},t),$$
$$\frac{\partial}{\partial t}\mathcal{G}(u,\zeta) = \left\{\exp\left[\mathcal{G}\left(u+\frac{\delta}{\delta\zeta};\xi\right) - \mathcal{G}(u;\xi)\right] - \exp\left(\mathcal{G}\left(\frac{\delta}{\delta\zeta};\xi\right)\right)\right\}\int dxu(\mathbf{x})\,L(\mathbf{x};\zeta) +$$
$$+\frac{1}{2}\int d\mathbf{x}'d\mathbf{x}''u_a(\mathbf{x}')\,u_b(\mathbf{x}'')\,b_{ab}(\mathbf{x}''-\mathbf{x}')\,F(u,\zeta). \tag{16}$$

Let us consider the simplest case — linear development of pair correlation only. So the dynamic equations (16) are linearized near some equilibrium state $\zeta_a^{(0)}$:

$$\frac{\partial \delta\zeta_a(\mathbf{x},t)}{\partial t} = \int d\mathbf{x}'T_{ab}(\mathbf{x},\mathbf{x}')\,\delta\zeta_b(\mathbf{x}') + \frac{1}{2}\int d\mathbf{x}'d\mathbf{x}''T_{abc}(\mathbf{x},\mathbf{x}',\mathbf{x}'')\,\xi_{bc}(\mathbf{x}',\mathbf{x}'',t) + \delta Y_a(\mathbf{x},t),$$
$$\frac{\partial \xi_{ab}(\mathbf{x}_1,\mathbf{x}_2,t)}{\partial t} = \int d\mathbf{x}'\xi_{ac}(\mathbf{x}_1,\mathbf{x}',t)\,T_{bc}(\mathbf{x}_2,\mathbf{x}') + \int d\mathbf{x}'\xi_{bc}(\mathbf{x}_2,\mathbf{x}',t)\,T_{ac}(\mathbf{x}_2,\mathbf{x}') + g_{ab}(\mathbf{x}_1,\mathbf{x}_2,t), \tag{17}$$
$$\hat{T}_{ab}(\mathbf{x},\mathbf{x}') \equiv \left.\frac{\delta L_a(\mathbf{x};\zeta)}{\delta\zeta_b(\mathbf{x}')}\right|_{\zeta=\zeta^{(0)}}, \quad \hat{T}_{abc}(\mathbf{x},\mathbf{x}',\mathbf{x}'') \equiv \left.\frac{\delta^2 L_a(\mathbf{x};\zeta)}{\delta\zeta_b(\mathbf{x}')\,\delta\zeta_c(\mathbf{x}'')}\right|_{\zeta=\zeta^{(0)}}.$$

Here $\delta\zeta_a(\mathbf{x},t)$ is the divergence of description parameters from the equilibrium state $\zeta_a^{(0)}$. We consider only pair correlations of description parameters because the external random force has only pair correlations. So, the correlations are generated by the external random force directly, but the correlations of higher orders can be generated by pair correlations only in a non-linear way and can be neglected in a close to equilibrium case.

3. DYNAMIC EQUATIONS FOR SYSTEM OF PARTICLES INTERACTING WITH MEDIA

Now we can use the results obtained above for description of the specific system of particles interacting with hydrodynamic media. Derivation of evolution equations for such system in microscopic approach is considered in work [6]. In paper [7] the long hydrodynamic tails theory for such system is constructed. But here we consider more general case, when the particles can be multiplied or captured by media. This will make the model closer to real problems of neutron scattering in different media. The description parameters are temperature $\zeta_0(\mathbf{x},t) \equiv T(\mathbf{x},t)$, medium velocity $\zeta_i(\mathbf{x},t) \equiv v_i(\mathbf{x},t)$, medium mass density $\zeta_4(\mathbf{x},t) \equiv \rho(\mathbf{x},t)$ and particle density $\zeta_5(\mathbf{x},t) \equiv n(\mathbf{x},t)$. The phenomenological terms, describing multiplication and capture of particles (see for example [8]) are included in the equations. Value K represents the overall growth (when $K > 0$) or decay (when $K < 0$) of neutron density caused by the multiplication and capture effects, D_{eff} is the effective diffusion ratio, and value E describes the heating of the system by both multiplication and capture processes. Some small effects such as thermodiffusion of neutrons are neglected. The evolution equations are

$$\dot{T}(\mathbf{x},t) = -(\mathbf{v}\nabla)\,T - T\left(\frac{\partial p}{\partial T}\right)_\rho (\rho c_v)^{-1}(\nabla\mathbf{v}) - \frac{\kappa}{\rho c_v}\Delta T + En + Y_0(\mathbf{x},t),$$
$$\dot{v}_i(\mathbf{x},t) = -\frac{1}{\rho}(\mathbf{v}\nabla)\,v_i - \frac{\nabla p}{\rho} - \frac{1}{\rho}\nabla_i(nT) + \frac{\eta}{\rho}\Delta v_i + \frac{1}{\rho}\left(\frac{\eta}{3}+\zeta_{visc}\right)\nabla_i(\nabla\mathbf{v}) + Y_i(\mathbf{x},t), \tag{18}$$
$$\dot{\rho}(\mathbf{x},t) = -\nabla(\rho\mathbf{v}),$$
$$\dot{n}(\mathbf{x},t) = -\nabla(n\mathbf{v}) - D_{eff}\Delta n + Kn + Y_5(\mathbf{x},t).$$

In work [6] it was shown that presence of particles can affect the kinetic coefficients such as viscosity η or heat conductivity κ. Here we suppose that all the kinetic coefficients already include particle correction, i.e. $\eta = \eta_{media} + \eta_n * n$ (see [6] for details), and after linearizing we assume, that $\eta = \eta_{media} + \eta_n * n_0$. In order to do further calculations we use Fourier transformations of description parameters and correlation functions:

$$\delta\zeta_a(\mathbf{x},t) = \int d^3k \exp(i\mathbf{q}\mathbf{x})\,\delta\zeta_a(\mathbf{q},t). \tag{19}$$

Using these formulas one can easily rewrite equations (17) in Fourier form, and it will be done further, but first we need to study the linearized motion equation for description parameters without correlations:

$$\delta\dot{\zeta}_a(\mathbf{q},t) = T_{ab}(\mathbf{q})\,\delta\zeta_b(\mathbf{q},t), \tag{20}$$

and find the eigenvectors of evolution matrix $T_{ab}(\mathbf{q})$:

$$T_{ab}(\mathbf{q})\,V_b^{(\mu)}(\mathbf{q}) = \lambda^{(\mu)}(q)\,V_a^{(\mu)}(\mathbf{q}). \tag{21}$$

The orthogonal normalized system of eigenvectors $V_a^{(\mu)}(\mathbf{q})$ where $\mu = 0,1,2,3,4,5$ will be used for construction of solutions of equations, containing evolution matrix $T_{ab}(\mathbf{q})$. The complete expression for this matrix is given by formula:

$$T_{ab}(\mathbf{q}) = \begin{pmatrix} -q^2\frac{\kappa}{\rho c_v} & -iq_k\frac{T}{\rho c_v}\left(\frac{\partial p}{\partial T}\right)_\rho & 0 & E \\ -iq_i\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_\rho & -\frac{\eta}{\rho}q^2\delta_{ik} - \frac{1}{\rho}\left(\frac{\eta}{3}+\zeta\right)q_iq_k & -iq_i\frac{1}{\rho}\left(\frac{\partial\rho}{\partial\rho}\right)_T & 0 \\ 0 & -iq_k\rho & 0 & 0 \\ 0 & -iq_kn & 0 & K - D_{eff}q^2, \end{pmatrix} \quad (22)$$

and it has quite complicated structure, so that explicit solution of the eigenvector problem (21) can not be found. Hence we use perturbation theory over small $\mathbf{q}$ and small K in order to obtain approximate solution. The first order approximation for evolution matrix $T^{(1)}_{ab}(\mathbf{q})$ is the part of the evolution matrix linear over $\mathbf{q}$. The second order correction changes only the eigenvectors, not eigenvalues. A separate problem is orthogonality and fullness of obtained eigenvectors. It is important that scalar product $(U,V) = U^*_aG_{ab}V_b$ and matrix

$$G_{ab} = \begin{pmatrix} \frac{A}{BS^2} & 0 & 0 & \\ 0 & \frac{\delta_{ik}}{S^2} & 0 & 0 \\ 0 & 0 & \frac{C}{BS^2} & 0 \\ 0 & 0 & 0 & \frac{T}{\rho NS^2} \end{pmatrix} \quad (23)$$

providing anti Hermitian character of matrix $\left(U, T^{(1)}(\mathbf{q})V\right) = \left(UT^{(1)}(\mathbf{q}), V\right)$ can be constructed, so its eigenvector system is orthogonal and complete automatically. In (23) new symbols are introduced:

$$A = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_\rho, \; B = \frac{T}{\rho c_v}\left(\frac{\partial p}{\partial T}\right)_\rho, \; C = \frac{1}{\rho}\left(\frac{\partial\rho}{\partial\rho}\right)_T. \quad (24)$$

After calculation of second order corrections (similar calculations are discussed in [7]) we obtain the eigenvectors and eigenvalues:

$$\begin{aligned} &V^{(0)}(q) = \sqrt{\frac{B\rho}{AC}}(-C,0,A,0), \\ &V^{(1,2)}(q) = \sqrt{\frac{1}{2}}(B, \mp St_k, \rho, n_0), \\ &V^{(3,4)}(q) = S\left(B, e_k^{(1,2)}, \rho, n_0\right), \\ &V^{(5)}(q) = \sqrt{\frac{nT}{\rho C}}\left(-2B, 0, \gamma\rho, \frac{\rho S^2}{T}\right), \end{aligned} \quad (25)$$

$$\begin{aligned} &\lambda^{(0)}(\mathbf{q}) = -\frac{\kappa q^2}{\gamma\rho c_v}, \; \lambda^{(1,2)}(\mathbf{q}) = \mp iqS - q^2\Gamma_S, \\ &\lambda^{(3,4)}(\mathbf{q}) = -\frac{\eta q^2}{\rho}, \; \lambda^{(5)}(\mathbf{q}) = K - D_{eff}q^2. \end{aligned} \quad (26)$$

In the latter the following value:

$$S^2 = AB + C\rho + \frac{nT}{\rho}, \quad (27)$$

represents sound velocity in the system, and

$$\begin{aligned} \Gamma_S = \frac{1}{2\rho}\Bigg\{&\frac{\kappa(\gamma-1)}{\gamma c_v} + \frac{4}{3}\eta + \zeta_{visc} + \\ &+ \frac{n_0T}{S^2}(D_{eff}+K) + \frac{\rho AEn}{S^2}\Bigg\} \end{aligned} \quad (28)$$

represents sound decay coefficient.

Let us now consider the simplest spatially homogeneous case, which is at equilibrium at $t = 0$:

$$\zeta^{(0)} = \text{const}, \quad \delta\zeta_a(\mathbf{x}, 0) = 0. \quad (29)$$

The stability of initial state can be provided by condition:

$$\text{Re}\left(\lambda^{(\mu)}(\mathbf{k})\right) \le 0, \quad \mu = 0, ..., 5 \quad (30)$$

for all values of $\mathbf{k}$. The external random force correlations and the correlators of hydrodynamic parameters are expected to be spatially homogeneous:

$$\begin{aligned} &\xi_{ab}(\mathbf{x}_1, \mathbf{x}_2, t) = \xi_{ab}(\mathbf{x}_1 - \mathbf{x}_2, t), \\ &g_{ab}(\mathbf{x}_1, \mathbf{x}_2, t) = g_{ab}(\mathbf{x}_1 - \mathbf{x}_2, t). \end{aligned} \quad (31)$$

Fourier transform of correlation functions reads:

$$\xi_{ab}(\mathbf{q}_1, \mathbf{q}_2, t) = \xi_{ab}(\mathbf{q}_1, t)\,\delta(\mathbf{q}_1 + \mathbf{q}_2), \quad (32)$$

and the evolution equations are quite simple:

$$\dot{\xi}_{ab}(\mathbf{q}, t) = \xi_{ac}(\mathbf{q}, t)\,T_{bc}(-\mathbf{q}) + \xi_{bc}(-\mathbf{q}, t)\,T_{ac}(-\mathbf{q}) + g_{ab}(\mathbf{q}), \quad (33)$$

$$\delta\dot{\zeta}_a(\mathbf{q}, t) = T_{ab}(\mathbf{q})\,\delta\zeta_b(\mathbf{q}, t) + \delta(\mathbf{q})\int d\mathbf{k}T_{abc}(\mathbf{k})\,\xi_{bc}(\mathbf{k}, t). \quad (34)$$

Solving the equation (33) we come up with the result:

$$\delta\dot{\zeta}_a(\mathbf{q}, t) = T_{ab}(\mathbf{q})\,\delta\zeta_b(\mathbf{q}, t) + \delta(\mathbf{q})\,\mathrm{Z}_a(t), \quad (35)$$

$$\mathrm{Z}_a(t) = \frac{1}{2}T_{abc;ij}\sum_{\mu=0}^{5}\sum_{\nu=0}^{5}\int d^3\mathbf{k}\frac{k_ik_j\left(1 - \mathrm{e}^{t\left(\lambda^{(\mu)}(\mathbf{k}) + \lambda^{(\mu)}(-\mathbf{k})\right)}\right)}{\lambda^{(\mu)}(\mathbf{k}) + \lambda^{(\mu)}(-\mathbf{k})}F^{\mu\nu}_{bc;ef}(\mathbf{k})\,g_{eff}(\mathbf{k}), \quad (36)$$

$$F^{\mu\nu}_{ab;cd}(\mathbf{k}) = V^{(\mu)}_a(\mathbf{q})\,V^{(v)}_b(-\mathbf{q})\,V^{*(\mu)}_{a'}(\mathbf{q})\,V^{*(v)}_{b'}(-\mathbf{q})\,G_{a'c}G_{b'd}. \quad (37)$$

The explicit expressions for $T_{abc}(\mathbf{k})$ and $T_{abc;ij}$ can be found from equation system (18) using definition (17). Expressions (35)-(37) show us that correlations of external random force result in additional effect on the system, that shifts the equilibrium point from the state $\zeta^{(0)}$ where it would be without correlations. More interesting case occurs when the initial state is unstable, i.e. equality (30) is wrong at least for one $\lambda^{(\sigma)}$. For example if $K > 0$, according to (26) we will have positive $\lambda^{(5)}$. If we consider this case without correlations we would have exponential growth proportional to $\exp(\lambda^{(5)}t)$ for hydrodynamic parameters. But according to (36) some components of correlations contribution $Z_a(t)$ grow twice faster by the law $\exp(2\lambda^{(5)}t)$, so correlations may accelerate instability development. Therefore, when we consider some medium under external neutron flux which is near equilibrium, presence of correlations in the flux (but not variation of neutron flux density) can affect the equilibrium state of the system.

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О ДЛИННОВОЛНОВЫХ ФЛУКТУАЦИЯХ В СИСТЕМАХ ЧАСТИЦ, ВЗАИМОДЕЙСТВУЮЩИХ С ГИДРОДИНАМИЧЕСКИМИ СРЕДАМИ

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Предложен стохастический подход к описанию длинноволновых флуктуаций в системах частиц, взаимодействующих с гидродинамическими средами. Наличие длинноволновых флуктуаций в системе обусловлено стохастичностью внешней случайной силы. Описание эволюции длинноволновых флуктуаций проводится с помощью метода усреднения нелинейных динамических уравнений по случайной внешней силе. Получены уравнения динамики длинноволновых флуктуаций обусловленных внешней случайной силой в гидродинамических средах с поглощением и размножением частиц. Рассмотренная система может служить моделью переноса нейтронов в гидродинамических средах. Обсуждается вопрос о влиянии флуктуаций на устойчивость стационарных состояний в системе.

ПРО ДОВГОХВИЛЬОВІ ФЛУКТУАЦІЇ В СИСТЕМАХ ЧАСТИНОК, ЩО СЛАБКО ВЗАЄМОДІЮТЬ ІЗ ГІДРОДИНАМІЧНИМИ СЕРЕДОВИЩАМИ

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Запропоновано стохастичний підхід до опису довгохвильових флуктуацій у системах частинок, що взаємодіють із гідродинамічними середовищами. Наявність довгохвильових флуктуацій у системі обумовлено дією випадкової зовнішньої сили. Опис еволюції довгохвильових флуктуацій здійснено за допомогою методу усереднення нелінійних динамічних рівнянь за стохастичною зовнішньою силою. Отримано рівняння динаміки довгохвильових флуктуацій, зумовлених зовнішньою випадковою силою в гідродинамічних середовищах з поглинанням та розмноженням частинок. Розглянута система може бути моделлю переносу нейтронів у гідродинамічних середовищах. Обговорено питання про вплив довгохвильових флуктуацій на стійкість стаціонарних станів в системі.