ON VERTICAL STRUCTURE OF ACCRETION DISKS IN TYPE IA SUPERNOVA PROGENITORS

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(Received November 21, 2011)

We study the vertical structure of a thin $\alpha-$disk [1] in a Type Ia supernova (SN Ia) progenitor system. Accurate equation of state and opacity of solar composition material are used. The results show pronounced features in the disk structure in the regions of ionization and molecule dissociation. We analyze the mass distribution in the disk, aiming to find the system parameters at which this mass distribution could explain the high velocity features (HVF) in SN Ia spectra [2]. We conclude that accretion disks are unlikely candidates for explaining the HVF.

PACS: 97.10.Gz, 97.60.Bw, 95.30.Lz, 44.40.+a, 44.25.+f, 42.25.Bs, 05.70.Ce, 05.20.-y, 02.70.Bf, 02.60.Lj

1. INTRODUCTION

Disk structures are ubiquitous in the Universe. They are formed when gas is drifted by gravitational force to the center of attraction, in the process decreasing its angular momentum by transporting it to the gas layers farther from the center via some viscosity mechanism. What is loosely called "gas" may in reality be plasma, it may contain solid particles, depending on the specific situation. Viscosity is believed to be turbulent/convective eddy viscosity, it may have magnetic origin; microscopic molecular viscosity is negligible to play any role. The heat being released by the viscosity, as well as other heat sources like irradiation by the central object, hot inner regions of the disk, other objects in the disk environment, is transferred to the disk surface and radiated into space. The heat production in the disk normally leads to the gas temperature increasing towards the disk equatorial plane; thermal pressure of the gas supports the gas vertical distribution (in $z-$direction, along the average gas momentum vector, perpendicular to the disk plane of symmetry), prevents it from collapsing to the equatorial plane.

Various phenomena in the disks, instabilities of the disks in their environment lead to intricate structure formation (galaxies, protoplanetary disks). Radiation from various disk regions is observed by astronomers, from subtle spectral features due to relatively dim protostellar and circumstellar disks, to active galactic nuclei and quasars, the brightest objects in the Universe, which are powered by accretion.

Different approximations are used for modeling different types of disk systems, or even different regions within the same disk, as certain effects important in one regime (e.g. radiation pressure, radial thermal energy advection, magnetic fields, disk self-gravity, irradiation by the nearby bright sources) are not important in the other ones. In this paper we study vertical structure of circumstellar disks, with parameters typical of the progenitor of type Ia supernova (SN Ia) system. Our interest is twofold:

1. Analyzing the role of accretion discs (AD) in high-velocity features (HVF) in the spectra of SN Ia, now considered ubiquitous [3,4]. HVF are absorption lines in the spectrum, in particular CaII infrared (IR) triplet near 800 nm, seen blueshifted at velocities $\sim 17\,000 ... 29\,000\ \mathrm{km\,s^{-1}}$ (up to 40 000 in SN 1994D), higher than the expansion velocity of the photosphere of the SN Ia ejecta. HVF are observed before, and up to ∼a week after SN Ia maximum light, then they fade. Several explanations were proposed for this feature. One generic model [2] not requiring Ca overproduction or any modifications to the now standard SN Ia scenarios is that these high-velocity lines are produced in a shell formed from circumstellar material (CSM) and outermost SN Ia ejecta as the former is overrun by the latter. The circumstellar material is assumed of standard solar composition.

This model was successfully tested by several groups against observations. It requires substantial mass of the shell: $5... 7 \times 10^{-3} M_{\odot}$ for SN 2005cg [4], $\sim 0.02 M_{\odot}$ for SN 2003du [2], $0.1 M_{\odot}$ for SN 1999ee [5], $0.2 M_{\odot}$ for SN 2005hj [6]. Large fraction of this mass originates from the CSM located in the vicinity of the SN Ia center, at distances $< 1.5 \times 10^{15}$ cm (for SN 2003du [2], from HVF timing considerations).

According to a single-degenerate scenario, SN Ia is a result of a thermonuclear explosion of a WD [7] that reached nearly Chandrasekhar mass ($M_{\mathrm{Ch}} \sim 1.38 M_{\odot}$) by accreting mass from a companion star, through Roche lobe overflow. Accretion disk (AD) is thus a natural candidate for the nearby mass.

2. In our simulations we use real equation of state (EOS) and opacities of solar composition gas [8–10],

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*E-mail address: azhiglo@kipt.kharkov.ua as the estimates of the disc mass found in the literature differ significantly, likely depending on the approximations used. As a result we are able to observe interesting features of the disk structure, in particular in the regions where H_2 molecules dissociate, where H and He ionize. While we only present our results for the specific value of the central object mass ($1.37\,M_\odot$, slightly under-Chandrasekhar WD) and a range of accretion rates relevant for SN Ia, similar features are observed for other disk parameters, and may have observational consequences. We integrate the structure equations to radii smaller than the WD radius (and a larger envelope around it formed as a result of accreted fuel burning on the WD surface) to spot all these features.

2. THEORY AND NUMERICAL METHOD

Circumstellar disks around compact objects with relatively low accretion rates are described by a thin $\alpha-$disk model [1]. Parameter α defines effective kinematic viscosity coefficient, $\nu = \alpha c_s H$, where c_s is the sound speed in the disk material, H is a characteristic scale height of the disk. This prescription assumes that characteristic turbulent velocity is αc_s, and characteristic turbulent eddy size is of order H. In original formulation [1] the vertical structure of AD (i.e. distribution of density ρ, pressure P, temperature T, heat flux F, etc. along vertical $z-$direction) was not studied in detail. Rough estimate $H = c_s/\Omega$ was made, where Ω is the Keplerian angular velocity at a given radius r in the disk. We use prescription

$$\nu = \epsilon \left(\frac{2r^3}{GM}\right)^{1/2} \frac{P}{\rho} \left[1 + \frac{2GM\rho}{Pr^3} z^2\right]^{-1/2} \qquad (1)$$

from [11] for direct comparison of the results. The square brackets here seem to take into account scale height (the height above the given point at which pressure changes by a factor of e) decreasing with z, due to increasing $g_z = GMz/(r^2 + z^2)^{3/2}$, the vertical component of gravity of the central object (nearly-Chandrasekhar WD in our study, $M = 1.37 M_\odot$). We used this prescription verbatim, albeit keeping these brackets is inconsistent with the thin disk approximation used, which neglects other effects $\sim (z/r)^2$.

It is claimed in [11] that relating ϵ to α as $\alpha = 3 \cdot 2^{-1/2} \epsilon$ yields description equivalent to original $\alpha-$model near the disk equatorial plane. We could not reproduce that estimate; instead we get $\epsilon = \alpha$ if H is defined as a scaleheight at $z = 0$. When original prescription [1] with $H = c_s/\Omega$ was used in our computations (it is used this way by many authors) we really got results close to those with recipe (1) with $\epsilon = \alpha$. We still define $\alpha - \epsilon$ correspondence below as $\alpha = 3 \cdot 2^{-1/2} \epsilon$ in agreement with [11]. $\alpha = 0.07$ is adopted as the base value, corresponding to $\epsilon = 1/30$ in [11]; values of α close to this are considered appropriate for circumstellar disks based on observations. It should be kept in mind that this may correspond to $\alpha = 0.033$ for the other definition of H used.

As $c_s \ll \Omega r$ radial pressure gradient is too small to support resting gas in radial graviatational field, $dp/dr \ll g_r \rho$. The gas thus rotates with almost Keplerian velocity, which by far exceeds its radial drift velocity, $v_r \ll v_\phi$. The heat is produced due to viscous friction between adjacent differentially rotating layers of gas; this determines vertical heat flux F:

$$\frac{dF}{dz} = \frac{9}{4} \frac{GM\nu\rho}{r^3}; \qquad (2)$$

radial heat flux is neglected in a thin disk. This heat is transported mainly by radiation and convection towards the disk photosphere, and is radiated into space. We use exact solution for the temperature gradient needed for transporting F in the upper radiative region of AD [12], in grey atmosphere approximation. At large optical depth $1 \ll \tau(z) \equiv \int_z^\infty \kappa\rho dz$ (Rosseland mean opacity κ is used) this simplifies to diffusion approximation often used [11]:

$$\left.\frac{dT}{dz}\right|_{rad} = \frac{-3\kappa\rho F}{16\sigma_{SB} T^3}, \qquad (3)$$

σ_{SB} being Stefan-Boltzmann constant. When the found pure radiative dT/dz exceeds adiabatic gradient Chandrasekhar instability drives convection, which transfers the heat in such regions; we set convective temperature gradient then as:

$$\left.\frac{dT}{dz}\right|_{conv} = -\gamma_2 g_z \rho \frac{T}{P}, \quad \gamma_2 = \left.\frac{\partial \ln T}{\partial \ln P}\right|_S. \qquad (4)$$

Rosseland mean opacities are taken from [9, 10] for solar composition disc [13]. Equation of state (EOS), including γ_2 are taken from [8].

We solve numerically a boundary-value problem for Eqs. (2)–(4) and $dp/dz = -g_z \rho$ with boundary conditions: $F|_{z=0} = 0$, $F|_{z=z_0} = F_0 = \frac{3GM\dot{M}}{8\pi r^3}$, values for $P_{z=z_0}$ and $T_{z=z_0}$ consistent with F_0 and $\tau_{z=z_0} = \tau_0$. τ_0 was fixed at 10^{-9}, z_0 was found by requiring $F|_{z=0} = 0$ when integrating down from z_0.

3. RESULTS

We varied accretion rate, shown in the figures if different from its default value of $\dot{M} = 10^{-6} M_\odot\,\mathrm{yr}^{-1}$; and α, the default value being $\alpha = 0.07$. We show the results for $\dot{M} \in [4 \times 10^{-8}; 2 \times 10^{-6}] M_\odot\,\mathrm{yr}^{-1}$ that covers (still controversial) rates typical of SN Ia progenitors [14–16]. α's in the range we study are used in the literature for various disk systems. Lower α values are seen to produce thicker and more massive disks. $\alpha = 0.01$ is used for T Tauri stars [17], $\alpha = 0.001$ was proposed for FU Orionis outbursts. However, larger values of $\alpha \sim 0.07$ and above are currently considered appropriate for circumstellar disks in post-main sequence binaries.

Fig. 1 shows profile of AD photosphere $z_\tau \equiv z_{\tau=2/3}$. It is plotted divided by r; $z_\tau/r = \mathrm{const}$ would correspond to conical disk shape. The inner thick disk region is radiation pressure dominated. At $r < 4 \times 10^9$ cm the disk is predominantly convective, in agreement with [11]. It is mostly radiative at $r \in [7 \times 10^9; 3 \times 10^{10}]$ cm; comparable proportions of radiative and convective regions are observed at $r \in [5\times 10^{10}; 1.2\times 10^{11}]$cm. The structure is more complex than in [11], likely due to varying opacity in this region due to lines of heavy elements accounted for in this work. The dip after $r \approx 1.2\times 10^{11}$ cm is due to HeIII recombining into HeII, as may be inferred from equatorial temperature profile in Fig. 2. The disk becomes convective abruptly, but contrary to [11] it becomes radiative back at $r \approx 4\times 10^{12}$cm (where hydrogen has become molecular, after the wide region where H and He recombine and H_2 molecules form); 3 more transitions occur at larger radii, as hydrogen γ_2 changes. The disk regains about a half of its pre-dip slope at $r \approx 10^{13}$cm; the slope declines slowly at larger radii through 10^{16}cm we integrated to. Similar features are observed for all $\dot{M}$ we tried.

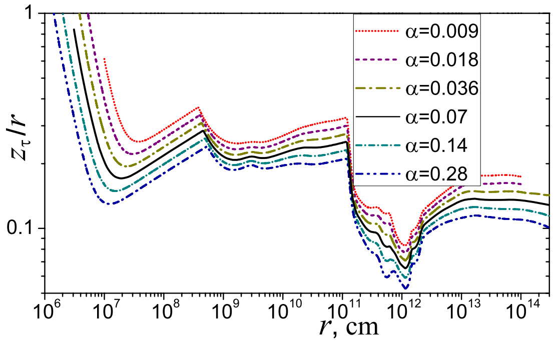

Fig. 1. *Integral slope of AD photosphere* z_τ/r*, at* $\dot{M} = 10^{-6} M_\odot\,\mathrm{yr}^{-1}$ *and various* α

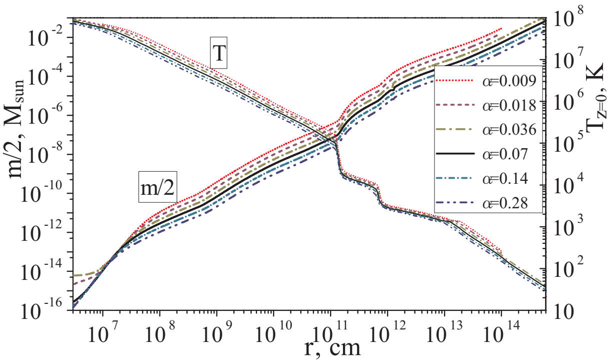

Fig. 2. *Equatorial temperature and half-mass of AD at* $\dot{M} = 10^{-6} M_\odot\,\mathrm{yr}^{-1}$*. Photospheric temperature* $T(z_\tau)$ *is independent of* α

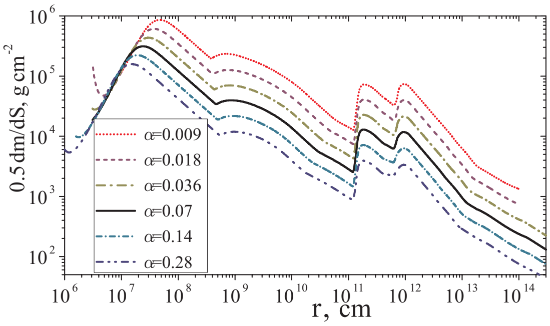

Fig. 3. *Half surface mass density of AD,* $1/2 \int_{-\infty}^{\infty} \rho(r,z)\,dz$ *as a function of radius, at* $\dot{M} = 10^{-6} M_\odot\,\mathrm{yr}^{-1}$

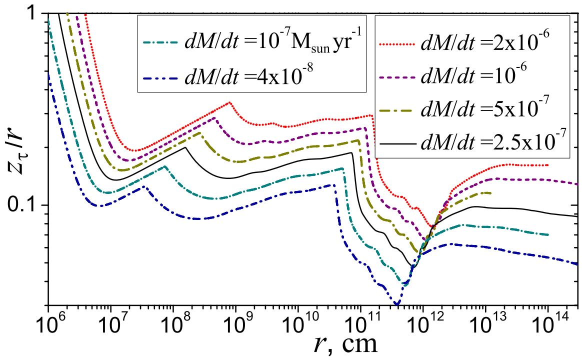

Fig. 4. *Dependence of* $z_\tau(r)/r$ *on accretion rate* dM/dt *(in units of* $M_\odot\,\mathrm{yr}^{-1}$*) at* $\alpha = 0.07$

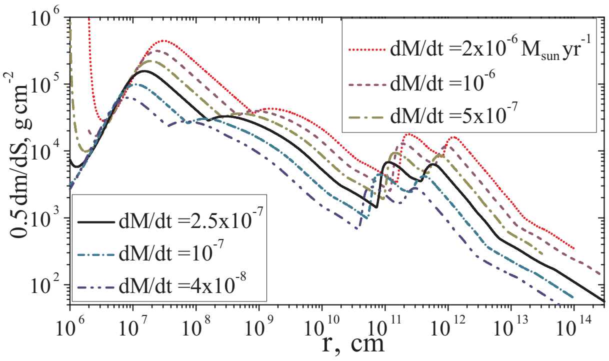

Fig. 5. *Half surface mass density of AD at various accretion rates* dM/dt. $\alpha = 0.07$

Fig. 3 shows surface mass density vs radius. The disk profile and surface density at $\alpha = 0.07$ and different accretion rates are shown in Figs. 4 and 5. 2D distributions of density and temperature in the inner part of the disk are shown in Figs. 6 and 7.

It is seen in Fig. 2 that AD radius must exceed ~ 3 AU for its mass to reach $0.01 M_\odot$. As the disk radius is smaller that that of the WD Roche lobe, this means that only largest supergiants (SG) could serve as the WD companions for such massive disks. It is still possible to observe sufficient line-of-sight mass density when watching close to the disk equatorial plane — at smaller radii. Fig. 8 shows anisotropy in mass distribution, for 3 radii chosen. Equivalent spherical CSM mass is $m_e(\theta) = 2dm/d\theta/\cos\theta$. It is seen that $m_e(0)$ (twice the value shown in Fig. 9) varies in $[4\times 10^{-3}; 9\times 10^{-2}] M_\odot$ for AD parameters shown, at $r_{\mathrm{AD}} \approx$1 AU. However, anisotropy of the mass distribution disagrees with observations [18].

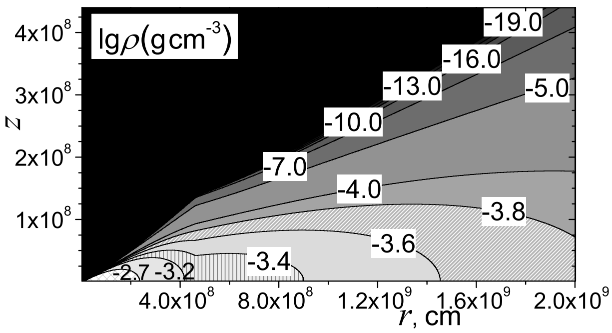

Fig. 6. *Density distribution in the inner part of AD.* $\alpha = 0.07$, $\dot{M} = 10^{-6} M_\odot\,\mathrm{yr}^{-1}$

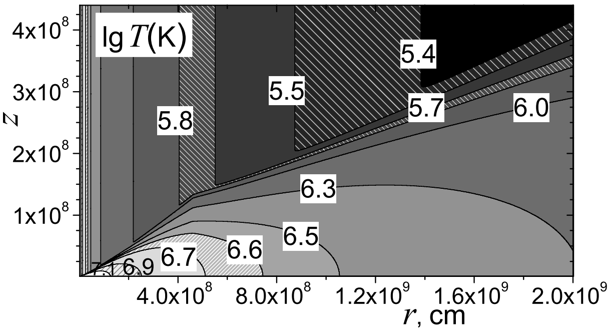

Fig. 7. *Temperature in the inner part of AD.* $\alpha = 0.07$, $\dot{M} = 10^{-6} M_\odot\,\mathrm{yr}^{-1}$

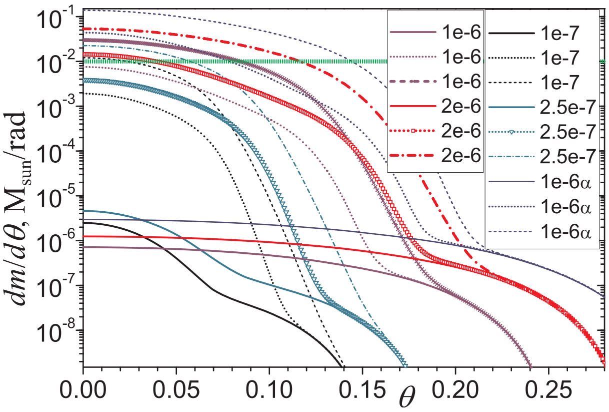

Fig. 8. *Angular density of the mass distribution. Angle θ of the line-of-sight is measured from the disk equatorial plane. Five sets of disk parameters are shown, differing in accretion rate, indicated in the legend at the respective curves (in units of $M_\odot\,\mathrm{yr}^{-1}$.) The group labeled "$1e-6\alpha$" corresponds to $\dot{M} = 10^{-6}$, $\alpha = 0.009$; in the rest of the models $\alpha = 0.07$. 3 curves are shown for each model, for disk radii 1.55×10^{11}, 1.55×10^{13} and 6.17×10^{13} cm. Smaller radii correspond to lower curves*

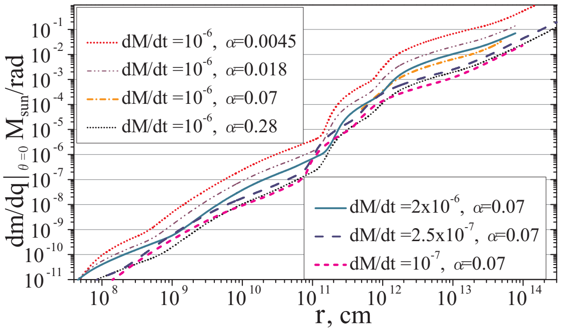

Fig. 9. *$dm/d\theta$ at $\theta = 0$ as a function of radius. Note that to get an inferred CSM mass of $10^{-2} M_\odot$ the disk radius must exceed 10^{12}cm even at very low $\alpha = 0.0045$. For the base model we study ($\alpha = 0.07$, $\dot{M} = 10^{-6}$) the disk radius should be 10^{13}cm, implying a SG companion*

4. DISCUSSION

The results for the disk mass distribution are thus hard to reconcile with observed ubiquity of HVF, assuming ADs the main source of the CSM mass required by impact model [2]. The disks must have substantial radii to have sufficient projected mass density even in their equatorial plane. Disks in binaries with Main Sequence stars are too light by a factor of $10^3 \ldots 10^4$. Only SG companions seem to allow for disks of sufficient radius; such larger disks are pronouncedly flatter, $dm/d\theta$ drops by a factor of 10 within $\sim 6°$ off equatorial $\theta = 0$ plane. Such mass distribution is at odds with current observations [18]. Significantly lower α would alleviate both of these discrepancies.

Better treatment of convection would alter the results somewhat. Temperature gradient would increase, so would the disk mass density; the disk would become flatter at the same time. The difference should not change the conclusions qualitatively, as comparison with radiative regions suggests.

The dust was assumed in thermal and hydrostatic equilibrium with gas. If it settles down to $z = 0$ fast in radiative regions that would decrease opacity, thus the temperature gradient.

The disk self-gravity was neglected, as the total disk mass does not exceed a few hundredth of solar mass. Gravity of the companion was found to alter the layers above AD photosphere substantially, but not have much effect on the inner layers and total disk mass.

AD irradiation by the WD and its companion was ignored. The bump in the disk photosphere at $r \approx 1.2 \times 10^{11}$ cm is high enough to shield the outer regions from the central object radiation; irradiation effect is insignificant at the bump and at smaller radii. The radiation from the bump (with surface $T \approx 10^4$ K) and from the companion star is intercepted by the disk. Irradiation usually makes disks lighter and geometrically thicker [17]; AD mass is still strongly concentrated towards $z = 0$.

It is unlikely that disk effective opening angle may be made larger due to impact with ejecta, as the latter runs through the disk with highly supersonic velocity. The outer, massive disk layers might broaden due to strong radiation from the growing SN Ia photosphere, before being hit by the ejecta. The *total* disk mass would then have to be of order $0.01 M_\odot$ to lead to the observed HVF; this requires yet smaller α or more extended disk. Under no circumstances a disk in a binary system with a main-sequence star as a WD companion is a plausible candidate for the circumstellar mass sufficient to explain the HVF via mechanism [2]. High temperatures of the gas in the disk at distances of order a few million kilometers would lead to H or He lines visible in early SN Ia spectra, contrary to observations. What the origin of the circumstellar mass in WD–MS binaries is (or whether HVF have some other, not CSM origin) thus remains an open question, requiring further theoretical and observational study.

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О ВЕРТИКАЛЬНОЙ СТРУКТУРЕ АККРЕЦИОННЫХ ДИСКОВ В ПРЕДШЕСТВЕННИКАХ СВЕРХНОВЫХ ТИПА Ia

А.В. Жигло

Мы изучаем вертикальную структуру тонкого α-диска [1] в системе предшественника сверхновой типа Ia (СН Ia). Использованы точное уравнение состояния и коэффициенты непрозрачности вещества солнечного состава. Результаты демонстрируют выраженные особенности в структуре диска в областях диссоциации молекул и ионизации. Анализируется распределение массы в диске с целью найти параметры системы, при которых это распределение могло бы объяснить высокоскоростные особенности (ВСО) в спектрах СН Ia [2]. Мы делаем вывод, что классические аккреционные диски являются маловероятными кандидатами для объяснения ВСО.

ПРО ВЕРТИКАЛЬНУ СТРУКТУРУ АКРЕЦІЙНИХ ДИСКІВ У ПОПЕРЕДНИКАХ НАДНОВИХ ТИПУ Ia

А.В. Жигло

Ми вивчаємо вертикальну структуру тонкого α-диску [1] в системі попередника наднової типу Ia (НН Ia). Застосовані точне рівняння стану і непрозорість речовини сонячного складу. Результати демонструють виражені особливості в структурі диску в областях дисоціації молекул та іонізації. Аналізується розподіл маси в диску з метою знайти параметри системи, при яких цей розподіл міг би бути поясненням особливостей з великими швидкостями (ОВШ) в спектрах НН Ia [2]. Ми робимо висновок, що класичні акреційні диски не є правдоподібними кандидатами для пояснення ОВШ.