STRING FRAGMENTATION MODEL WITH QUARK SPIN

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A string model of quark hadronization, taking the quark spin degree of freedom into account, is proposed. The method for using the model in a Monte-Carlo code for jet generation is given.

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1. INTRODUCTION

In *deep inelastic collisions,* high energy, quasi-free *partons* are produced in short distance ($\ll 1$ fm) subprocesses, then are indirectly detected as hadron jets. The transition *partons* $\rightarrow$ *jets*, called hadronization, occurs at long distance ($\gtrsim 1$ fm) by creation of new quark-antiquark pairs. The pairs arrange themselves in chains connecting the initially produced partons in a way such that each jet is color neutral. Fig. 1 depicts such a chain in e^+e^- annihilation or $W^\pm$ decay into quark q_A + antiquark $\bar{q}_B$ and no gluon. We restrict ourselves to processes without baryon production.

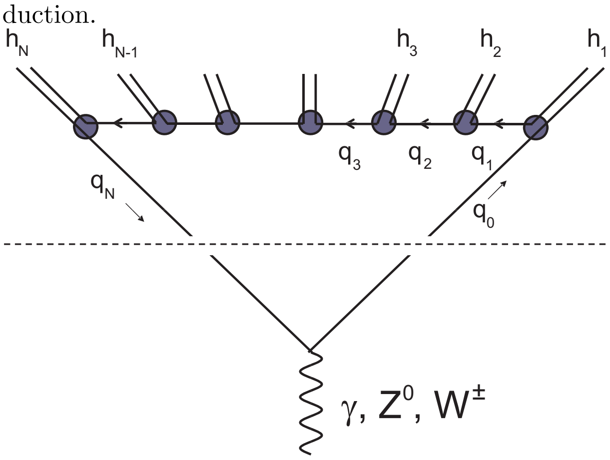

Fig. 1. *e^+e^- annihilation or $W^\pm$ decay in quark-antiquark $\rightarrow$ hadrons*

The recursive model. Looking at the upper part of Fig. 1 from right to left, the hadronisation process

$$q_A + \bar{q}_B \rightarrow h_1 + h_2 ... + h_N \tag{1}$$

can be described as the recursive process [1, 2]

$$\begin{aligned} q_0 &\rightarrow h_1 + q_1 \ , \\ q_1 &\rightarrow h_2 + q_2 \ , \\ &\cdots\cdots \\ q_{N-1} &\rightarrow h_N + q_B \ . \end{aligned} \tag{2}$$

n is the *rank* of the hadron h_n. $q_0 \equiv q_A$ and $q_B \equiv q_N$ is the charge conjugate of $\bar{q}_B$ propagating "backward in time" with 4-momentum $q_B \equiv -\bar{q}_B$. The conservation of 4-momentum holds at each step:

$$q_{n-1} = p_n + q_n \ , \qquad 1 \leq n \leq N, \tag{3}$$

where p_n is the hadron 4-momentum. q_n designates either the quark species or its 4-momentum. In the simplest recursive model, the sharing between p_n and q_n in (3) is made according to the *splitting probability distribution*:

$$d\zeta_n \, d^2\mathbf{q}_{n\mathrm{T}} \, f(\zeta_n, q_{n\mathrm{T}}) \ , \tag{4}$$

provided the remaining mass square $(q_n + \bar{q}_B)^2$ is still large. $\mathbf{q}_\mathrm{T} = (q^x, q^y)$, $\zeta_n = q_n^+/q_{n-1}^+$ and $q^\pm \equiv q^0 \pm q^z$. $\mathbf{q}_\mathrm{A}$ and $\bar{\mathbf{q}}_\mathrm{B}$ define the $+z$ and $-z$ directions.

Including the quark *flavor* degree of freedom, is relatively easy. The splitting function for $q \rightarrow h + q'$ depends on the flavors and writes $f_{q',h,q}(\zeta, q'_\mathrm{T})$.

In a straightforward way the recursive model lends itself to the Monte-Carlo method for the simulations of jets. Such simulations are essential in the preparation and analyzis of any high-energy experiment.

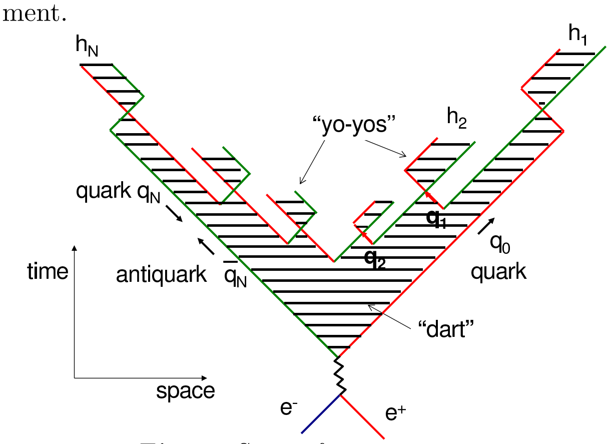

Fig. 2. *String fragmentation*

The string fragmentation model [3, 4]. Hadronization of Fig. 1 is considered as the cascade decay of a massive string stretching between q_A and $\bar{q}_B$, called *dart*. The space-time picture is shown in Fig. 2. At

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the n^{th} string breaking point (starting from the right) a $q_n\bar{q}_n$ pair is created. $\bar{q}_n$ moves to the right, meets q_{n-1} which is moving to the left and both form the hadron h_n, which is a short string and, in a pure classical model, oscillates like a yo-yo. If the null-plane coordinate $X^- = t - z$ is used as time variable, the hadrons are emitted in the ordering of (2) and the string model can be treated as a recursive one, with the *symmetric Lund* [4] splitting function,

$$f_{q',h,q}(\zeta, \mathbf{q}'_\mathrm{T}, \mathbf{q}_\mathrm{T}) \tag{5}$$
$$\propto Z^{a\{q\}}\ (1/Z-1)^{a\{q'\}}\ \exp\left(-b\,\frac{m_h^2+\mathbf{p}_\mathrm{T}{}^2}{Z}\right),$$

where $Z = 1-\zeta$ and $a\{q\} \equiv a_q(\mathbf{q}_\mathrm{T}^2)$ depends in principle on the quark flavor q and transverse momentum $\mathbf{q}_\mathrm{T}$. Note the dependence of f in both $\mathbf{q}_\mathrm{T}$ and $\mathbf{q}'_\mathrm{T}$. Eq. (5) is used in the Monte-Carlo simulation code PYTHIA, with a unique value a for $a\{q\}$.

The string fragmentation model is invariant under:

– (a) rotations about the z-axis;
– (b) Lorentz transformations along the z-axis;
– (c) mirror reflection about any plane containing the z-axis (equivalent to parity);
– (d) *quark line reversal* or "left-right symmetry", *i.e.,* interchanging the roles of q_A and $\bar{q}_B$.

The model is not covariant *locally* (step by step), but *global* covariance holds for the whole process of Fig. 1.

Role of the quark spin. There is experimental evidence that the quark spin plays a dynamical role in jet formation.

– A natural $\mathbf{p}_\mathrm{T}$-dependent polarization of inclusive hyperons has been observed for a long time. Most proposed mechanisms involve the creation of a polarized strange quark at $\mathbf{q}_\mathrm{T} \neq 0$.
– The **Collins effect** [5], yielding a transversely polarized quark fragmentation function of the form (Fig. 3)

$$F(z, \mathbf{p}_\mathrm{T}\,; \mathbf{S}) = F_0(z, \mathbf{p}_\mathrm{T}^2)\ [1 + A_\mathrm{T}\, \mathbf{S}_\mathrm{T}.(\hat{\mathbf{z}} \times \mathbf{p}_\mathrm{T})/|\mathbf{p}_\mathrm{T}|]\,, \tag{6}$$

has been established in semi-inclusive deep inlelastic lepton-hadron scattering [6]. The *analyzing power* $A_\mathrm{T} = A_{q_0,h}(z, p_\mathrm{T})$ cannot be calculated in perturbative QCD but could in principle be fully determined by experiments.
– Another asymmetry, **jet handedness** [7, 8], has been predicted for a longitudinally polarized quark. Selecting, for instance, the 3 fastest hadrons of the jet, h, h' and h'', there should be a correlation between the sign of the helicity and the sign of $\mathbf{p}\cdot(\mathbf{p}' \times \mathbf{p}'')$.

Collins effect for *transversity* and jet handedness for *helicity* will provide complementary *quark polarimeters*. A theoretical model of jets with spinning quarks will be usefull to optimize their analyzing powers.

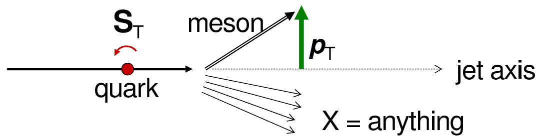

Fig. 3. *Single-particle Collins effect*

Inclusion of the spin degree of freedom. The aim of this paper is to build a recursive model with quark spin effects, not only for the initial quark but along the whole quark line of Fig. 1. Translating it in a Monte-Carlo algorithm is not trivial. In the case of flavor, it suffices to draw lots for the species u, d or s of the new quark at each step of (2). If we do the same for quark *helicity*, we miss the pure *transversity* states, which are linear combinations of helicity states, and vice versa.

We will proceed in four steps. In Section 2 we review the classical "string + $^3P_0$" mechanism [4, 9] giving transverse spin effects. In Section 3 we present the covariant *quark-multiperipheral* model using Dirac spinors. A reduction to Pauli spinors and an ultra-simplified version [10] is reviewed in Section 4. Finally, in Section 5, a semi-quantized string model with Pauli spinors is presented and the corresponding Monte-Carlo algorithm is described in Section 6.

Notations. The symbol $\{q_n\}$, in curly brackets, *e.g.* in Eq. (7), represents the momentum *and* the flavor of the n^{th} quark altogether. A four-momentum q is separated in its transverse part $\mathbf{q}_\mathrm{T} = (q^x, q^y)$ and time-longitudinal part $q_\mathrm{L} = (q^0, q^z)$. For the latter we also use the null-plane coordinates $q^\pm = q^0 \pm q^z$. The virtual mass square is $q^2 = q_\mathrm{L}{}^2 - \mathbf{q}_\mathrm{T}{}^2 = q^+q^- - \mathbf{q}_\mathrm{T}{}^2$.

The polarization vector of a quark is decomposed as $\mathbf{S} = (S_\mathrm{L}, \mathbf{S}_\mathrm{T})$ where $S_\mathrm{L}/2$ = helicity, $\mathbf{S}_\mathrm{T}$ = transversity. The associated density matrix is $\rho = (1 + \mathbf{S}.\vec{\sigma})/2$, with $\mathbf{S}^2 \leq 1$.

2. THE CLASSICAL *STRING* + $^3P_0$ MECHANISM OF COLLINS EFFECT [9]

We consider the simplest case where all the emitted partices are pseudoscalar mesons. Then $(q_n\bar{q}_{n-1})$ is a spin singlet. At a string breaking the $q_n\bar{q}_n$ pair is assumed to be created in the $^3P_0$ state with zero total momentum (corresponding to the vacuum quantum numbers). Fig. 4 depicts the recursive decay of the dart when q_0 has a transverse, anti-clockwise polarization. $(q_0\bar{q}_1)$ is a spin-singlet, therefore $\bar{q}_1$ spins clockwise. $(q_1\bar{q}_1)$ is a spin-triplet, therefore q_1 spins also clockwise. Due to the $^3P_0$ configuration, the relative $q_1 - \bar{q}_1$ orbital momentum $\mathbf{L}_1$ is opposite to the spins, therefore anti-clockwise. It makes $\bar{q}_1$ move upward and q_1 move downward in the figure. The upward momentum of q_1 is taken by hadron h_1, resulting in a Collins effect, $\mathbf{p}_{1\mathrm{T}}$ being on the side of $\mathbf{S}_{0\mathrm{T}} \times \hat{\mathbf{z}}$.

Iterating this reasoning, q_2 and $\bar{q}_2$ are spinning anti-clokwise, $\mathbf{L}_2$ is clokwise, etc. It gives to h_2, h_3, etc. Collins effects of alternate sides. Of course, successive spins are not so rigidly coupled and the Collins effect decays along the quark chain. Nevertheless the model predicts a Collins effect for h_2 opposite to that of h_1 and reinforced by the fact that q_1 and $\bar{q}_2$ move on the same side. This is in agreement with experiment.

The classical string + ${}^3P_0$ mechanism also explains the polarization of inclusive hyperons [4].

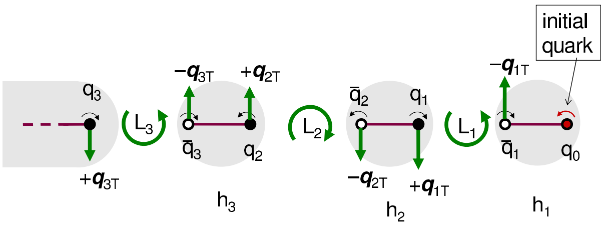

Fig. 4. *The string + ${}^3P_0$ mechanism for Collins effect*

3. THE COVARIANT QUARK-MULTIPERIPHERAL MODEL

The upper half of Fig. 1 looks like a multiperipheral diagram [11], but with quark instead of meson exchanges. We treat q_A and $\bar{q}_B$ as on mass-shell quarks and assume that the probability of the whole process of Fig. 1 factorizes in the probabilities of the upper and lower parts. The amplitude of (1) writes

$$\begin{aligned}&\mathcal{M}\{q_A\bar{q}_B \to h_1 h_2 ... h_N\}\\ &= \Gamma\{q_B, h_N, q_{N-1}\}\,\Delta\{q_{N-1}\}\cdots \\ &\cdots\Delta\{q_2\}\,\Gamma\{q_2,h_2,q_1\}\,\Delta\{q_1\}\,\Gamma\{q_1,h_1,q_A\}\,.\end{aligned} \tag{7}$$

$\Delta\{q\} = D_q(q^2)\,(\mu_q + \gamma.q)$ is the quark propagator. μ_q is the quark mass. $D_q(q^2)$ is a fast decreasing function of $|q^2|$. $\Gamma\{q',h,q\} \equiv \Gamma_{q',h,q}(q',q)$ is the $q \to h + q'$ vertex function, which is a 4×4 matrix in the space of Dirac spinors. For a pseudoscalar meson, $\Gamma\{q',h,q\} = \gamma_5\, G_{q',h,q}({q'}^2, q^2)$. The model is covariant *locally*, *i.e.,* at each vertex and propagator.

Another important approximation is to neglect interferences between several diagrams which lead to the same final state. Then the total hadronisation cross section writes (omitting the flux factor)

$$\begin{aligned}&\sigma\{\bar{q}_B, q_A\} = \sum_N \sum_{h_1,...h_N} \int \frac{\mathrm{d}^3\mathbf{p}_1\cdots\mathrm{d}^3\mathbf{p}_N}{p_1^0\cdots p_N^0}\\ &\times\ \delta^4(p_1+p_2...+p_N-q_A-\bar{q}_B\}\\ &\times |\bar{v}(\bar{q}_B,\mathbf{S}_\mathrm{B})\,\mathcal{M}\{q_A\bar{q}_B\to h_1h_2...h_N\}\,u(q_A,\mathbf{S}_\mathrm{A})|^2\,.\end{aligned} \tag{8}$$

The second summation bears on the hadron species. $u(q_A,\mathbf{S}_\mathrm{A})$ and $v(\bar{q}_B,\mathbf{S}_\mathrm{B})$ are the Dirac spinors of q_A and $\bar{q}_B$.

4. REDUCTION TO Pauli SPINORS

We now describe the spin degree of freedom in the most economical way, with Pauli instead of Dirac spinors. We give up local covariance, but maintain the invariances (a), (b), (c) and (d) listed in the introduction about the string model. For this we replace [10]

- $u(q_0,\mathbf{S}_0)$ by the Pauli spinor $\chi(\mathbf{S}_0)$
- $\bar{v}(q_{\bar{q}_B},\mathbf{S}_{\bar{q}_B})$ by $-\chi^\dagger(-\mathbf{S}_{\bar{q}_B})\,\sigma_z$
- γ_5 by σ_z
- $\mu_q + \gamma.q$ by $\mu_q + \sigma_z\,\sigma.\mathbf{q}_\mathrm{T}$.

We also give up covariance for propagators. Thus,

$$\Delta\{q\} = D_q({q_\mathrm{L}}^2, {\mathbf{q}_\mathrm{T}}^2)\ (\mu_q + \sigma_z\,\sigma.\mathbf{q}_\mathrm{T})\ . \tag{9}$$

An ultra-simplified model [10]. We consider only pseudo-scalar mesons, with a momentum-independent emission vertex σ_z, and take a factorized, flavor-independent quark propagator

$$\Delta\{q\} = D_L({q_\mathrm{L}}^2)\ \exp(-B{\mathbf{q}_\mathrm{T}}^2/2)\ (\mu + \sigma_z\,\sigma.\mathbf{q}_\mathrm{T})\,. \tag{10}$$

Furthermore we ignore the mass-shell constraint

$$\begin{aligned}m_n^2 = (q_{n-1}^+ - q_n^+)(q_{n-1}^- - q_n^-)\\ -(\mathbf{q}_{n-1,\mathrm{T}} - \mathbf{q}_{n,\mathrm{T}})^2.\end{aligned} \tag{11}$$

This crude approximation achieves the full decoupling of the longitudinal momenta from the transverse ones and from the quark spin. The joint $\mathbf{p}_T$ distibutions of the n first mesons have simple expressions, for instance

$$\begin{aligned}I(\mathbf{p}_{1\mathrm{T}},\mathbf{p}_{2\mathrm{T}},\mathbf{p}_{3\mathrm{T}}) \propto \exp(-B\mathbf{q}_{\mathrm{T}1}^2 - B\mathbf{q}_{\mathrm{T}2}^2 - B\mathbf{q}_{\mathrm{T}3}^2)\\ \times\,\mathrm{Tr}\left\{\mathbf{M}_3\,\mathbf{M}_2\,\mathbf{M}_1\,\rho_0\,\mathbf{M}_1^\dagger\,\mathbf{M}_2^\dagger\,\mathbf{M}_3^\dagger\right\},\end{aligned} \tag{12}$$

where $\rho_0 = (1+\mathbf{S}_0.\sigma)/2$ is the spin density matrix of q_0 and $\mathbf{M}_n = (\mu + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}n})\,\sigma_z$. For complex μ one has a Collins effect for each meson, the analyzing power depending only on the meson rank. For h_1,

$$A_T = 2\frac{\mathrm{Im}(\mu)\,|\mathbf{p}_{1,T}|}{|\mu|^2 + \mathbf{p}_{1,T}^2}\,. \tag{13}$$

The above ultra-simplified model is a kind of theoretical laboratory. It reproduces the transverse spin effects of the classical string + ${}^3P_0$ mechanism and predicts jet handedness. It can be extended to mesons of nonzero spins [10].

5. THE SEMI-QUANTIZED STRING MODEL

Let us first consider spinless quarks and mesons. Following the sum-over-histories approach of Feynman, to the classical string history of Fig. 2 we associate the amplitude

$$\begin{aligned}&\mathcal{M}(q_A\bar{q}_B \to h_1h_2...h_N) = \exp\left[(-i\kappa_C + 2i\kappa)\,\mathcal{A}\right]\\ &\times\left(q_A^+p_1^-\right)^{\alpha\{q_A\}} \times (-p_1^+p_2^- - i0)^{\alpha\{q_1\}}\ldots\\ &\cdots(-p_{N-1}^+p_N^- - i0)^{\alpha\{q_{N-1}\}} \times (p_N^+\bar{q}_B^{\,-})^{\alpha\{q_B\}}\\ &\times g\{q_B,h_N,q_{N-1}\}\cdots g\{q_2,h_2,q_1\}\,g\{q_1,h_1,q_0\}\,.\end{aligned} \tag{14}$$

- $\mathcal{A}$ is the space-time area swept by the dart. $\kappa_C = \kappa - i\mathcal{P}/2$ is the *complex* string tension of the dart [12], accounting for its unstability (in analogy with the complex mass $m - i\Gamma/2$ of an unstable particle). The *string fragility* $\mathcal{P}$ is the probability of string breaking per unit of space-time area swept by the string. $\kappa \simeq 1$ GeV/fm, $\mathcal{P}/\kappa \sim 10^{-1}$. We will use $b \equiv \mathcal{P}/(2\kappa^2)$.

The exponent of the first line contains the pure *string action* of the dart (proportional to $-\kappa C$) and "missing propagation phases" (proportional to 2κ) of the final hadrons, taking into account their different emission points [13].

- The first and last power-law factors of the 2nd and 3rd lines take into account the *quark actions* of q_A and $\bar{q}_B$, which in the case of non-zero mass follow the pieces of hyperbolas in Fig. 5. We have

$$\alpha\{q_A\} = (b - i/\kappa)\,\mu_A^2/2 \quad \text{(idem for } \bar{q}_B\text{)}. \qquad (15)$$

These factors also take into account the "missing string area" between the hyperbolas and the broken-line trajectories that would be followed by massless quarks.

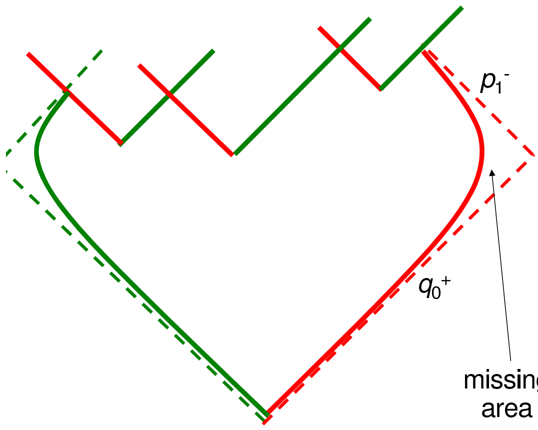

Fig. 5. *Trajectories of massive q_A and $\bar{q}_B$*

- The intermediate power-law factors take into account the actions of the quarks and antiquarks pairs created at string ruptures. One may take the analytic continuations of (15), replacing q_A^+ by $-p_n^+$, p_1^- by p_{n+1}^- :

$$\alpha\{q_n\} = (\mu_n^2 + \mathbf{q}_{n\mathrm{T}}^2)\,(b - i/\kappa)/2\,. \qquad (16)$$

For real μ_n the modulus square of the n^{th} factor is

$$(p_n^+ p_{n+1}^-)^{b(\mu_n^2+\mathbf{q}_{n\mathrm{T}}^2)} \exp[-\pi(\mu_n^2 + \mathbf{q}_{n\mathrm{T}}^2)/\kappa]\,, \qquad (17)$$

which contains the characteristic exponential factor of Schwinger tunneling [4]. In Fig. 6 we have represented tunneling by a dotted line linking the classical hyperbolic trajectories of q_n and $\bar{q}_n$.

There is however a limitation to Eq. (16). The tunneling length $2E_\mathrm{T}/\kappa = 2(\mu^2 + \mathbf{q}_\mathrm{T}^2)^{1/2}/\kappa$ must be smaller than the string length which is of the order of $\mathcal{P}^{-1/2}$. As a matter of fact the production of large E_T quarks is not in the domain of the string model, but of perturbative QCD. Besides, $\Re\mathrm{e}\,\alpha\{q_n\}$ should not be too large. One may cure this requirement by setting $b = 0$ in Eq. (16).

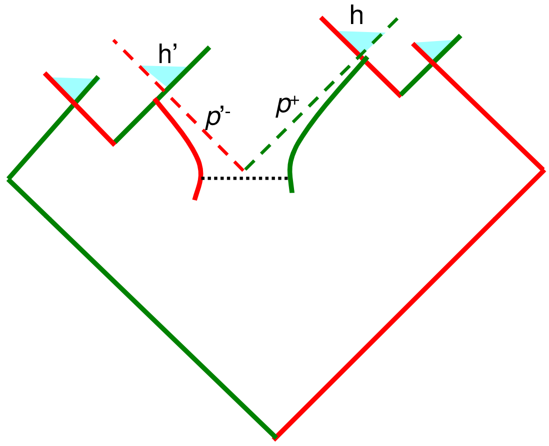

Fig. 6. *Tunneling trajectories of q_n and $\bar{q}_n$*

- The last line contains vertex functions

$$g\{q', h, q\} \equiv g_{q',h,q}({\mathbf{q}_\mathrm{T}'}^2, \mathbf{q}_\mathrm{T}'.\mathbf{q}_\mathrm{T}, {\mathbf{q}_\mathrm{T}}^2) \qquad (18)$$

which depend on flavours and *transverse* momenta, but not on longitudinal momenta. Quark line reversal imposes g to be symmetric under the interchange $\{q; \mathbf{q}_T\} \leftrightarrow \{q'; \mathbf{q}_T'\}$.

Expressing the fully differential cross section of (1) as the modulus square of (14) for the one recovers the symmetric Lund model [4].

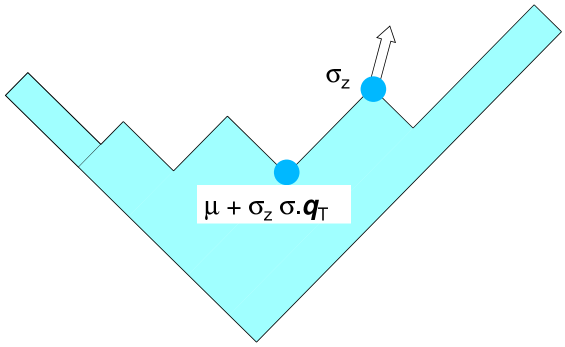

Fig. 7. *Spin matrices to b inserted in the string amplitude*

Inclusion of quark spin. Spin is simply included by inserting the 2×2 matrices of the ultra-simplified model of Section 4. Fig. 7 indicates where such matrices operate. Restricting ourselves to pseudoscalar meson production, we have to multiply the expression (14) by the chain of 2×2 matrices

$$\sigma_z\,(\mu_{N-1} + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}\,N-1})\,\sigma_z \cdots$$
$$\cdots(\mu_2 + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}\,2})\,\sigma_z\,(\mu_1 + \sigma_z\sigma.\mathbf{q}_{\mathrm{T}\,1})\,\sigma_z\,. \qquad (19)$$

To sum up, the fully differential cross section of (1) with polarized q_A and $\bar{q}_B$ is given by

$$\sigma\{\bar{q}_B, q_A\} = \sum_N \sum_{h_1,\ldots h_N} \int \mathrm{d}^4 q_1 \cdots \mathrm{d}^4 q_{N-1}$$
$$\times\, 2\delta(p_1^2 - m_1^2) \cdots 2\delta(p_N^2 - m_N^2)$$
$$\times\, \left|\chi^\dagger(-\mathbf{S}_\mathrm{B})\,\sigma_z\,\mathcal{M}\,\chi(\mathbf{S}_\mathrm{A})\right|^2, \qquad (20)$$

$\mathcal{M}$ being given by (14) times (19).

Unlike the ultra-simplified model of [10], the present string fragmentation model takes into account the mass-shell conditions properly.

6. RECURSIVE MONTE-CARLO ALGORITHM

To apply Eq. (20) in Monte-Carlo simulations, we have to put the model in a recursive form. This will be done in 5 steps:
– re-express $\mathcal{M}$ as a multiperipheral amplitude,
– introduce the *cross section matrix* $\mathcal{R}$ of the reaction $q_n+\bar{q}_B \to$ *hadrons*, and write a recursion relation for it,
– write the Regge behavior of $\mathcal{R}$ at large $(q_n+\bar{q}_B)^2$,
– give the receipe for the Monte-Carlo generation of the 4-momentum q_n once q_{n-1} and the polarization $\mathbf{S}_{n-1}$ are known,
– write the formula giving $\mathbf{S}_n$ knowing $\mathbf{S}_{n-1}$, q_{n-1} and q_n.

In the following we use units such $\kappa = 1$, $\kappa_C = 1 - i\mathcal{P}/2 = 1 - ib$.

Multiperipheral form of $\mathcal{M}$. The string amplitude (14) times (19) can be put in a multiperipheral form. The *splitting amplitude*, which we define as the product of the $n^{\rm th}$ vertex and the $n^{\rm th}$ propagator, writes

$$\begin{aligned} T_n \equiv T\{q_n, h_n, q_{n-1}\} &= \Delta\{q_n\}\,\Gamma\{q_n, h_n, q_{n-1}\} \\ &= \exp\left[(i-b)\,(q^+_{n-1}p^-_n)/2\right] \\ &\times (q^+_{n-1}p^-_n)^{\alpha\{q_{n-1}\}}(-p^+_n/q^+_n - i0)^{\alpha\{q_n\}} \\ &\times g\{q', h, q\}\,(\mu_n + \sigma_z\sigma.\mathbf{q}_{n\mathrm{T}})\,\sigma_z\,. \end{aligned} \quad (21)$$

Introducing the sub-amplitude $\mathcal{M}_{N-n}$ for $q_n + \bar{q}_B \to h_{n+1} + \cdots h_N$, we have

$$\mathcal{M} \equiv \mathcal{M}_N = \mathcal{M}_{N-n}\,T_n \cdots T_2\,T_1. \quad (22)$$

The cross section matrix. Using (22), the n-particle inclusive cross section with polarized quarks writes

$$\begin{aligned} &\frac{d\sigma(q_A+\bar{q}_B \to h_1, \cdots h_n + X)}{d^3\mathbf{p}_1/p^0_1 \cdots d^3\mathbf{p}_n/p^0_n} \\ &= \mathrm{Tr}\{\,\rho_0\,T^\dagger_1\,T^\dagger_2 \cdots T^\dagger_n\,\mathcal{R}_n\,T_n \cdots T_2\,T_1\}\,, \end{aligned} \quad (23)$$

where ρ_0 is the spin density matrix of q_A and

$$\begin{aligned} \mathcal{R}(q_n) = &\sum_{N>n} \int \frac{\mathrm{d}^3\mathbf{p}_{n+1}\cdots \mathrm{d}^3\mathbf{p}_N}{p^0_{n+1}\cdots p^0_N} \\ &\times \mathcal{M}^\dagger_{N-n}\,\sigma_z\,\frac{1-\sigma.\mathbf{S}(\bar{q}_B)}{2}\sigma_z\,\mathcal{M}_{N-n} \end{aligned} \quad (24)$$

is the *cross section matrix* [14] of the reaction $q_n + \bar{q}_B \to$ *hadrons*. It operates in the space of the q_n spin states. It also depends on the antiquark polarization $\mathbf{S}(\bar{q}_B)$, but at large $(q_n+\bar{q}_B)^2$ this dependence is negligible and we may take $\mathbf{S}(\bar{q}_B) = 0$.

Fig. 8 represents the unitarity diagram giving the total $q_A + \bar{q}_B$ cross section matrix $\mathcal{R}(q_A)$ in the multiperipheral picture. Encircled in dashed line is the unitarity diagram for $\mathcal{R}(q_1)$. The general cross section matrix $\mathcal{R}\{q\}$ satisfies the integral recursion relation

$$\mathcal{R}\{q\} = \sum_h \int \frac{\mathrm{d}^3\mathbf{p}}{p^0}\,T^\dagger\{q', h, q\}\,\mathcal{R}\{q'\}\,T\{q', h, q\}\,. \quad (25)$$

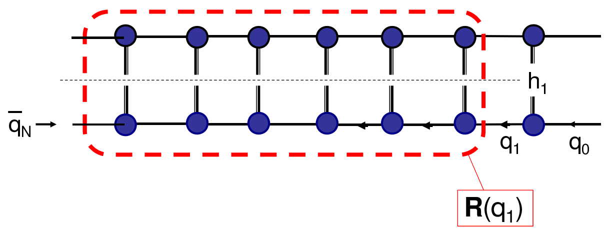

Fig. 8. *Unitarity diagram for $\mathcal{R}$*

Regge behavior of $\mathcal{R}\{q\}$. It is known that the unitarity diagrams of the multiperipheral model build up an *output reggeon*. Thus we assume a Regge behavior at large $(q+\bar{q}_B)^2$ for the cross section matrix:

$$\mathcal{R}\{q\} \sim |(\bar{q}_B)^- q^+|^{\alpha_{\rm out}}\,\left[\beta_q(\mathbf{q}^2_{\rm T}) + \gamma_q(\mathbf{q}^2_{\rm T})\,\sigma_z\sigma.\mathbf{q}_{\rm T}\right]. \quad (26)$$

A preliminary numerical task consists in calculating $\alpha_{\rm out}$ and the *Regge residue* functions, $\beta_q(\mathbf{q}^2_{\rm T})$ and $\gamma_q(\mathbf{q}^2_{\rm T})$, using Eq. (25).

Monte-Carlo generation of momenta. Suppose that we know the flavor and momentum of quark $\{q\} \equiv \{q_{n-1}\}$ and its polarization $\mathbf{S} \equiv \mathbf{S}_{n-1}$. From Eqs.(25) and (26) , one can write the $q + \bar{q}_B \to$ *hadrons* cross section as

$$\begin{aligned} \sigma\{q+\bar{q}_B\} &= \mathrm{Tr}\,\{\rho\,\mathcal{R}\{q\}\} = |(\bar{q}_B)^-|^{\alpha_{\rm out}} \sum_h \int \frac{\mathrm{d}^3\mathbf{p}}{p^0} \\ &\times ({q'}^+)^{\alpha_{\rm out}}\ \mathrm{Tr}\left[\,T\{q', h, q\}\,\rho\,T^\dagger\{q', h, q\}\right. \\ &\left.\times\ [\beta_{q'}({\mathbf{q}'_{\rm T}}^2) + \gamma_{q'}({\mathbf{q}'_{\rm T}}^2)\,\sigma_z\sigma.\mathbf{q}'_{\rm T}]\,\right] \end{aligned} \quad (27)$$

with $h + q' = q$ and $\rho = (1+\sigma.\mathbf{S})/2$. The expression in the last two lines is proportional to the probability that quark $\{q\} \equiv \{q_{n-1}\}$ emits a hadron $\{h_n\}$ of species h and 4-momentum p. Following the Monte-Carlo method, one generates h and p at random according to this probability. $\{q'\} \equiv \{q_n\}$ is then fixed by the conservation of charge, strangeness and 4-momentum.

Polarization of the next quark. Once the flavors and momenta of $\{p\} \equiv \{p_n\}$ and $\{q'\} \equiv \{q_n\}$ are known, the $\{q'\}$ polarization is given by

$$\frac{1+\sigma.\mathbf{S}'}{2} \equiv \rho' = \frac{T\{q', h, q\}\ \rho\ T^\dagger\{q', h, q\}}{\mathrm{Tr}\,(\ T\{q', h, q\}\ \rho\ T^\dagger\{q', h, q\}\ )}. \quad (28)$$

The trace is symbolically represented in Fig. 9 for the case $n = 1$.

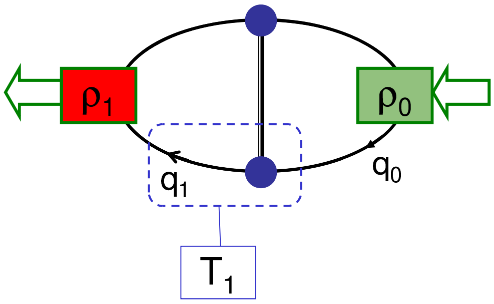

Fig. 9. *Loop diagram representing the trace in Eq. (28)*

7. CONCLUSIONS

We have given the principle of a recursive quark fragmentation model which includes the spin degree of freedom. Since the latter has an essentially quantum nature, we started from *amplitudes* rather *probabilities*.

The model can produce the spin asymmetries of Collins and jet handedness, if we give an imaginary part to the quark mass μ. For the moment we have no theoretical justification for that, but no argument against. In fact μ may not be the actual quark mass but some parameter of the nonperturbative physics of hadronization. Anyway, we do not claim that our model is closer to reality than other ones. It is just a first attempt to treat spin in a systematic manner in jet physics. Much work remain to be done to implement this model in a jet simulation code like PYTHIA.

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СТРУННАЯ МОДЕЛЬ ФРАГМЕНТАЦИИ, УЧИТЫВАЮЩАЯ СПИН КВАРКА

К. Артру

Предложена струнная модель адронизации кварка, учитывающая спиновую степень свободы кварка. Дан метод применения этой модели в программе Монте-Карло для генерации струн.

СТРУННА МОДЕЛЬ ФРАГМЕНТАЦІЇ, ЩО ВРАХОВУЄ СПІН КВАРКА

К. Артру

Запропоновано струнну модель адронізації кварка, що враховує спіновий ступінь свободи кварка. Дано метод застосування цієї моделі в програмі Монте-Карло для генерації струн.