# ON REGULAR GEONS IN GENERAL RELATIVITY

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We present the exact regular solution of vacuum Einstein equations, which may be interpreted as "mass without mass" (geon – in terminology of J.A. Wheeler), moving along a straight line with constant velocity. The feature of this classical object, localized in three-dimensional space, is that its scalar invariant constructed of two Riemann curvature tensors does not impose any restrictions on the velocity of the object.

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## 1. INTRODUCTION

Wheeler [1] was the first who suggested, that vacuum Einstein equations allow the existence of regular, localized in curved space-time classical objects – geons, such that the distant observer sees the curvature concentrated in the central region with persisting large-scale structure. However, various attempts to construct such objects as static or stationary curved space-time, possibly coupled to other zero-mass fields such as massless neutrinos or the electromagnetic field [2, 3], so far not been successful. In particular, these geons are however believed to be unstable, owing to the tendency of massless fields either to disperse to infinity or to collapse into a black hole [4].

In 1985, Sorkin [5] generalized Wheeler's geon into a topological geon by allowing that its central region may have complicated topology. This geon has a regular Euclidean-signature section, but the topologically nontrivial central region may evolve into black hole [6]. So, recent researches in this area are studying the properties of families of geon-like black holes in D $\geq$ 4 space-time dimensions [7].

## 2. THE REGULAR VAQUUM GEON

All the above mentioned objects, described by nonlinear differential equations, are static. This means that there exists a system of reference, in which parts of the structure under consideration are in rest relative to each other. It is interesting to consider the localized nonlinear object, in which separate parts move relative to one another. In the simplest case it can be assumed, that a three-region of a curved four-dimensional space-time is moving relative to another with a constant 3-velocity $\vec{v}\,(v,0,0)$

In such a space-time the spatial direction $x$ as well as the plane $(yz)$, perpendicular to it, are selected, so the space-time metric can be chosen in the form:

$$ds^2 = uc^2dt^2 - fdx^2 - \chi\left(dy^2 + dz^2\right) + 2\psi\, cdtdx, \quad (1)$$

where $u, f, \chi, \psi$ are functions of the parameter:

$$\xi = \sqrt{(x - vt)^2 + y^2 + z^2}.$$

Determinant of this metric tensor is

$$g = -(uf + \psi^2)\chi^2. \quad (2)$$

Vacuum Einstein equations:

$$R_{ik} = 0 \quad (3)$$

have the following exact solution:

$$\begin{aligned} u &= (1 - \frac{v}{c}a_4)^2 \frac{a_3^2}{a_2}\frac{(a_1-\xi)^2}{(a_1+\xi)^2} - \frac{v^2}{c^2}\frac{a_2}{\xi^4}(a_1+\xi)^4, \\ f &= -a_4^2\frac{a_3^2}{a_2}\frac{(a_1-\xi)^2}{(a_1+\xi)^2} + \frac{a_2}{\xi^4}(a_1+\xi)^4, \\ \chi &= \frac{a_2}{(a_1\xi)^4}(a_1+\xi)^4, \\ \psi &= \left(1 - \frac{v}{c}a_4\right)a_4\frac{a_3^2}{a_2}\frac{(a_1-\xi)^2}{(a_1+\xi)^2} + \frac{v}{c}\frac{a_2}{\xi^4}(a_1+\xi)^4, \end{aligned} \quad (4)$$

where $a_1, \ldots, a_4$ are the integration constants.

The scalar invariant, composed of two curvature tensors, for this space-time has the form:

$$I = \frac{1}{48}R_{iklm}R^{iklm} = \left(\frac{2\xi^3}{a_1a_2}\right)^2\left(1 + \frac{\xi}{a_1}\right)^{-12}. \quad (5)$$

In contrast to the Schwarzschild solution, for which this invariant is of the form:

$$I_{Schw} = \left(\frac{r_g}{2r^3}\right)^2, \quad (6)$$

where $r$ is the distance to the center of the object, $r_g$ is Schwarzschild radius, the invariant (5) is regular everywhere in the space-time: it goes to zero as $r^6$, when the center of the geon is approached, and behaves like $r^{-6}$ on large distances from it.

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### 3. CONCLUSIONS

We obtain exact solution for vacuum Einstein equations, which is asymptotically flat and regular everywhere in the space-time. The scalar invariant (5) as well as the determinant of the metric tensor (2) contains the velocity of the geon only through the parameter $\xi$. The integration constant $a_4$ is not included in (2) and (5) at all. As is seen from (5), this invariant does not include any limitations on the velocity of the geon. The question of stability of this object requires further study.

## References


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### О РЕГУЛЯРНЫХ ГЕОНАХ В ОБЩЕЙ ТЕОРИИ ОТНОСИТЕЛЬНОСТИ


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Представлено точное регулярное решение вакуумных уравнений Эйнштейна, которое можно интерпретировать как «массу без массы» (геон – по терминологии Дж.А. Уилера), движущуюся вдоль прямой с постоянной скоростью. Особенностью этого классического объекта, локализованного в трехмерном пространстве, является то, что его скалярный инвариант, составленный из двух тензоров Римана, не содержит каких-либо ограничений на скорость объекта.


### ПРО РЕГУЛЯРНІ ГЕОНИ У ЗАГАЛЬНІЙ ТЕОРІЇ ВІДНОСНОСТІ


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Представлено точне регулярне розв'язання вакуумних рівнянь Ейнштейна, яке можна інтерпретувати як «масу без маси» (геон – по термінології Дж.А. Уілера), що рухається вздовж прямої з постійною швидкістю. Особливістю цього класичного об'єкта, локалізованого у тривимірному просторі, є те, що його скалярний інваріант, складений з двох тензорів Рімана, не містить будь-яких обмежень на швидкість об'єкта.