ON UNIFIED THEORETICAL MODELS, DYNAMICAL TORSION AND SPIN

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Unified field theoretical models based on generalized affine geometries are proposed, described and analyzed from the physical and mathematical points of view. The relation between torsion and spin of the models is explicitly shown and some of their physical consequences, as the modification of the anomalous momentum of the elementary particles, are discussed.

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1. THE THEORY

In this report the geometrical analysis of a new type of Unified Field Theoretical (UFT) models introduced previously in [1, 2] by the authors is presented. These new unified theoretical models are characterized by an underlying hypercomplex structure, zero non-metricity and the geometrical action is determined fundamentally by the curvature arising thanks to the breaking of symmetry of a group manifold in higher dimensions. This mechanism of Cartan-MacDowell-Mansouri type, permits us to construct geometrical actions of determinantal type. Such mechanism also leads to a non topological physical Lagrangian due to the splitting of a reductive geometry. The starting point of the type of theory is a space-time basis Manifold equipped with a metric, e.g.

$$M, g_{\mu\nu} \equiv e_\mu \cdot e_\nu, \tag{1}$$

where for each point $p \in M$ $\exists$ a local space affine A. The connection over A $\widetilde{\Gamma}$ defines a generalized affine connection Γ on M specified by (∇, K) where K is an invertible $(1,1)$ tensor over M. We will demand that the connection is compatible and rectilinear

$$\nabla K = KT, \ \nabla g = 0, \tag{2}$$

where T is the torsion, and g (the space-time metric used to raise and to low indices and determines the geodesics) is preserved under parallel transport. This generalized compatibility condition ensures that the affine generalized connection Γ maps autoparallels of Γ on M in straight lines over the affine space A (locally). The first equation in (2) is equal to the condition determining the connection in terms of the fundamental field in the UFT non-symmetric. For instance, K can be identified with the fundamental tensor in the non-symmetric fundamental theory. This fact gives us the possibility to restrict the connection to an (anti) Hermitian theory.

The second important point is the following: let us consider [1] the extended curvature

$$\mathcal{R}^{ab}_{\mu\nu} = R^{ab}_{\mu\nu} + \Sigma^{ab}_{\mu\nu} \tag{3}$$

with

$$R^{ab}_{\mu\nu} = \partial_\mu \omega^{ab}_\nu - \partial_\nu \omega^{ab}_\mu + \omega^{ac}_\mu \omega_{\nu c}{}^{b} - \omega^{ac}_\nu \omega_{\mu c}{}^{b}, \tag{4}$$

$$\Sigma^{ab}_{\mu\nu} = -\left(e^a_\mu e^b_\nu - e^a_\nu e^b_\mu\right).$$

We assume here ω^{ab}_ν a $SO(d-1,1)$ connection and e^a_μ is a vierbein field. The eqs. (3) and (4) can be obtained, for example, using the formulation that was pioneering introduced in seminal works by E. Cartan long time ago [1]. It is well known that in such a formalism the gravitational field is represented as a connection of one form associated with some group which contains the Lorentz group as subgroup. The typical example is provided by the $SO(d,1)$ de Sitter gauge theory of gravity. In this specific case, the $SO(d,1)$ the gravitational gauge field $\omega^{AB}_\mu = -\omega^{BA}_\mu$ is broken into the $SO(d-1,1)$ connection ω^{ab}_μ and the $\omega^{da}_\mu = e^a_\mu$ vierbein field, with the dimension d fixed. Then, the de Sitter (anti-de Sitter) curvature

$$\mathcal{R}^{AB}_{\mu\nu} = \partial_\mu \omega^{AB}_\nu - \partial_\nu \omega^{AB}_\mu + \omega^{AC}_\mu \omega_{\nu C}{}^{B} - \omega^{AC}_\nu \omega_{\mu C}{}^{B} \tag{5}$$

splits in the curvature (3). At this point, our goal is to enlarge the group structure of the space-time Manifold of such manner that the curvature (5), obviously after the breaking of symmetry, permits us to

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define the geometrical Lagrangian of the theory as

$$L_g = \sqrt{det\mathcal{R}^a{}_\mu \mathcal{R}_{a\nu}} = \sqrt{detG_{\mu\nu}}\,. \qquad (6)$$

Then the action is

$$S = \frac{b^2}{4\pi}\int \sqrt{-g}dx^4\mathbb{R},$$

$$\mathbb{R} \equiv \sqrt{\gamma^4 - \frac{\gamma^2}{2}\overline{G}^2 - \frac{\gamma}{3}\overline{G}^3 + \frac{1}{8}\left(\overline{G}^2\right)^2 - \frac{1}{4}\overline{G}^4},$$

$$G_{\mu\nu} \equiv \lambda^2\left(g_{\mu\nu} + f^a{}_\mu f_{a\nu}\right) + \qquad (7)$$
$$+2\lambda\left(R_{(\mu\nu)} + f^a{}_\mu R_{[a\nu]}\right) + R^a{}_\mu R_{a\nu},$$

$$G^\nu_\nu \equiv \lambda^2\left(d + f_{\mu\nu}f^{\mu\nu}\right) + \qquad (8)$$
$$+2\lambda\left(R_S + R_A\right) + \left(R_S^2 + R_A^2\right)$$

with (the upper bar on the tensorial quantities indicates traceless condition, b^2 in principle constant homogenizing the units)

$$\begin{array}{ll} R_S \equiv g^{\mu\nu}R_{(\mu\nu)}, & R_A \equiv f^{\mu\nu}R_{[\mu\nu]}, \\ & \overline{G}^\nu_\rho\overline{G}^\rho_\nu \equiv \overline{G}^2, \\ \gamma \equiv \frac{G^\nu_\nu}{d}, & \overline{G}_{\mu\nu} \equiv G_{\mu\nu} - \frac{g_{\mu\nu}}{4}G^\nu_\nu, \\ \overline{G}^\nu_\lambda\overline{G}^\lambda_\rho\overline{G}^\rho_\nu \equiv \overline{G}^3, & \left(\overline{G}^\nu_\rho\overline{G}^\rho_\nu\right)^2 \equiv \left(\overline{G}^2\right)^2, \\ & \overline{G}^\nu_\mu\overline{G}^\mu_\lambda\overline{G}^\lambda_\rho\overline{G}^\rho_\nu \equiv \overline{G}^4, \end{array} \qquad (9)$$

where the variation was made with respect to the assumed for $f_{\rho\tau}$ (electromagnetic) potential a_τ as follows:

$$R_{(\mu\nu)} = \overset{\circ}{R}_{\mu\nu} - T_{\mu\rho}{}^\alpha T_{\alpha\nu}{}^\rho = -\lambda g_{\mu\nu}, \qquad (10)$$

$$R_{[\mu\nu]} = \nabla_\alpha T^\alpha_{\mu\nu} = -\lambda f_{\mu\nu}, \qquad (11)$$

$$\frac{\delta\sqrt{G}}{\delta a_\tau} = \nabla_\rho\left(\frac{\partial\sqrt{G}}{\partial f_{\rho\tau}}\right) \equiv \nabla_\rho\mathbb{F}^{\rho\tau} =$$
$$= \nabla_\rho\left[\frac{\lambda^2N^{\mu\nu}\left(\delta^\sigma_\mu\ f^\rho{}_\nu + \delta^\sigma_\nu\ f^\rho{}_\mu\right)}{2\mathbb{R}}\right] = 0. \qquad (12)$$

From this set, the link between the torsion T and f will be determined. Notice that total antisymmetry of the torsion is assumed due to the physical consequences that it property for T carry up. Also, f is not a priori potential for the torsion T and $N^{\mu\nu}$ is a tensor coming from the variational procedure and defined in [1, 2]. Our goal is to take advantage of the geometrical and topological properties of this theory in order to determine the minimal group structure of the resultant space-time Manifold able to support a fermionic structure. From this fact, the relation between antisymmetric torsion and Dirac structure of the space-time is determined and the existence of an important contribution of the torsion to the gyromagnetic factor of the fermions as follows [1, 2, 5]:

$$\left[\left(\widehat{P}_\mu - e\widehat{A}_\mu\right)^2 - m^2 - \frac{1}{2}e\sigma^{\mu\nu}F_{\mu\nu}\right]u^\lambda + \qquad (13)$$
$$+\frac{1}{2}\sigma^{\mu\nu}R^\lambda_{\rho[\mu\nu]}u^\rho - \frac{1}{2}e\sigma^{\mu\nu}\left(\widehat{A}_\mu\widehat{P}_\nu - \widehat{A}_\nu\widehat{P}_\mu\right)u^\lambda = 0.$$

It is interesting to see that

i) the above formula is absolutely general for the type of geometrical Lagrangians involved containing the generalized Ricci tensor inside,

ii) for instance, the variation of the action will carry the symmetric contraction of components of the torsion tensor, then the arising of terms as $h_\mu\ h_\nu$ (h_ν is an axial vector dual of the torsion).

iii) the only thing that changes is the mass (see [2]) and the explicit form of the tensors involved as $R^\lambda_{\rho[\mu\nu]}$, $F_{\mu\nu}$, etc. is without variation of the Dirac general structure of the equation under consideration,

iv) eq. (13) differs from the one obtained by Landau and Lifshitz by the appearance of last two terms: the term involving the curvature tensor is due to the spin interaction with the gravitational field (due to the torsion term in $R^\lambda_{\rho[\mu\nu]}$) and the last term is the spin interaction with the the electromagnetic and the mechanical momenta,

v) expression (13) is valid for another vector v^λ, then is valid for a bispinor of the form $\Psi = \mathbf{u} + i\mathbf{v}$,

vi) the meaning for a quantum measurement of the space-time curvature is mainly due to the term in (13) involving explicitly the curvature tensor.

The important point here is that the spin-gravity interaction term is so easily derived as the spinors are represented as space-time vectors whose covariant derivatives are defined in terms of the G-(affine) connection. In their original form the Dirac equations would have, in curved space-time, their momentum operators replaced by covariant derivatives in terms of "spin-connection" whose relation is not immediately apparent.

2. DIRAC STRUCTURE, ELECTROMAGNETIC FIELD AND ANOMALOUS GYROMAGNETIC FACTOR

The interesting point now is based on the observation that if we introduce expression (11) in (13) then:

$$\left[\left(\widehat{P}_\mu - e\widehat{A}_\mu\right)^2 - m^2 - \frac{1}{2}e\sigma^{\mu\nu}F_{\mu\nu}\right]u^\lambda -$$
$$-\frac{\lambda}{d}\frac{1}{2}\gamma^5\sigma^{\mu\nu}f_{[\mu\nu]}u^\lambda - \frac{1}{2}e\sigma^{\mu\nu}\left(\widehat{A}_\mu\widehat{P}_\nu - \widehat{A}_\nu\widehat{P}_\mu\right)u^\lambda = 0, \qquad (14)$$

$$\left[\left(\widehat{P}_\mu - e\widehat{A}_\mu\right)^2 - m^2 - \frac{1}{2}\sigma^{\mu\nu}\left(eF_{\mu\nu} + \gamma^5\frac{\lambda}{d}f_{\mu\nu}\right)\right]u^\lambda - \qquad (15)$$
$$-\frac{e}{2}\sigma^{\mu\nu}\left(\widehat{A}_\mu\widehat{P}_\nu - \widehat{A}_\nu\widehat{P}_\mu\right)u^\lambda = 0.$$

We can see clearly that if $\widehat{A}_\mu = ja_\mu$ (with j being an arbitrary constant), $F_{\mu\nu} = jf_{\mu\nu}$ the last expression takes the suggestive form:

$$\left[\left(\widehat{P}_\mu - e\widehat{A}_\mu\right)^2 - m^2 - \frac{1}{2}\left(ej + \gamma^5 \frac{\lambda}{d}\right)\sigma^{\mu\nu} f_{\mu\nu}\right] u^\lambda - \\ -\frac{e}{2}\sigma^{\mu\nu}\left(\widehat{A}_\mu \widehat{P}_\nu - \widehat{A}_\nu \widehat{P}_\mu\right) u^\lambda = 0 \tag{16}$$

with the result that the term corresponding to the gyromagnetic factor has been modified to $\left(j + \gamma^5 \frac{\lambda}{ed}\right)/2$. Notice that in an Unified Theory, with the characteristics introduced here, is reasonable the identification introduced in the previous step ($F \leftrightarrow f$) in order that the fields arise from the same geometrical structure (as is possible due to the $\mathrm{U}(2, C)$ fundamental structure of the space-time, main ingredient of the arising of matter from the geometrical structure of the Manifold).

The concrete implications about this important contribution of the torsion to the gyromagnetic factor will be given elsewhere with great detail on the dynamical property of the torsion field. We remark only the following:

i) there exists an important contribution of the torsion to the gyromagnetic factor that can have implicancies to the trouble of the anomalous momentum of fermionic particles,

ii) this contribution appears (taking the second equality of expression (11)), as a modification on the vertex of interaction, almost from the effective point of view;

iii) it is quite evident that this contribution will probably justify the little appearance of the torsion at great scale, because we can bound the torsion due to the other well known contributions to the anomalous momenta of the elementary particles (QED, weak, hadronic contribution, etc),

iv) the form of the coupling spin-geometric structure coming from the first principles, as the Dirac equation, not prescriptions,

v) then, from iii) how the covariant derivative works in presence of torsion is totally determined by the G structure of the space-time,

vi) the Dirac equation (14) (where the second part of the equivalence (11) was introduced coming from the equation of motion), said us that the vertex was modified without a dynamical function of propagation. Then, other form to see the problem treated in this paragraph is to introduce the propagator for the torsion corresponding to the first part of the equivalence (11) written as follows:

$$\left[\left(\widehat{P}_\mu - e\widehat{A}_\mu\right)^2 - m^2 - \frac{1}{2}e\sigma^{\mu\nu}F_{\mu\nu}\right] u^\lambda - \\ -\frac{1}{2d}\gamma^5\sigma^{\mu\nu}\nabla_\mu h_\nu u^\lambda - \\ -\frac{1}{2}e\sigma^{\mu\nu}\left(\widehat{A}_\mu \widehat{P}_\nu - \widehat{A}_\nu \widehat{P}_\mu\right) u^\lambda = 0, \tag{17}$$

where the relation between the totally antisymmetric torsion tensor and its dual h_μ has been used. Analyzing the first two terms of expression (17) we can guess the explicit form of the vertex and the effective Dirac Lagrangian where the new modification comes from:

$$\mathcal{L}_{feff} = \overline{\Psi}\left(i\gamma^\mu\partial_\mu - m + \gamma^5\gamma^\mu h_\mu + e\gamma^\mu A_\mu\right)\Psi.$$

Then, the formal Feynman propagator that we are looking for is evidently such:

$$S(p) = \left(i\gamma^\mu p_\mu \gamma^\mu - m + \gamma^5\gamma^\mu h_\mu + e\gamma^\mu A_\mu\right)\cdot\frac{\Delta}{\Lambda},$$

where we have defined

$$\Delta \equiv \left(p^2 - m^2 + h^2 + e^2A^2\right) + \\ + 2\left[\gamma^5\left(p\cdot h + m\gamma^\mu h_\mu\right) + e\left(p\cdot A + em\gamma^\mu A_\mu\right)\right],$$

$$\Lambda \equiv \left(p^2 - m^2 + h^2 + e^2A^2 + i\epsilon\right)^2 - \\ - 4\left[\left(p\cdot\left(\gamma^5 h + eA\right)\right)^2 - m^2\left(h^2 + e^2A^2\right)\right].$$

This important possibility including several processes of interest in modern physics (anomalous momentum of the muon, velocity of the neutrino, neutrino spin flip, etc) will be studied soon elsewhere [5].

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References

1. D.J. Cirilo-Lombardo // *Int. J. Theor. Phys.* 2010, v. 49, p. 1288-1301 (and references therein).

2. D.J. Cirilo-Lombardo // *Int. J. Theor. Phys.* DOI 10.1007/s10773-011-0678-1 (and references therein).

3. D.J. Cirilo-Lombardo // *Class. Quantum Grav.* 2005, v. 22, p. 4987-5004 (and references therein).

4. P. Singh // *Class. Quantum Grav.* 1990, v. 7, p. 2125.

5. D.J. Cirilo-Lombardo and S.N. Shulga, in preparation.

О МОДЕЛЯХ ЕДИНОЙ ТЕОРИИ ПОЛЯ, ДИНАМИЧЕСКИХ КРУЧЕНИЯХ И СПИНЕ

Д.Х. Сирило-Ломбардо

Модели единой теории поля, основанные на обобщенных аффинных геометриях, предложены, описаны и проанализированы с физико-математической точки зрения. Явно показана связь между кручением и спином в этих моделях. Обсуждаются некоторые их физические следствия, например, модификация аномального импульса элементарных частиц.

ПРО МОДЕЛІ ЄДИНОЇ ТЕОРІЇ ПОЛЯ, ДИНАМІЧНІ КРУТІННЯ І СПІН

Д.Х. Сіріло-Ломбардо

Моделі єдиної теорії поля, засновані на узагальнених афінних геометріях, запропоновані, описані й проаналізовані з фізико-математичної точки зору. Явно показаний зв'язок між крутінням і спином у цих моделях. Обговорюються деякі їхні фізичні наслідки, наприклад, модифікація аномального імпульсу елементарних часток.