THRESHOLD ELECTRON-POSITRON PAIR PRODUCTION BY A POLARIZED ELECTRON IN A STRONG MAGNETIC FIELD

O.P. Novak*and R.I. Kholodov

Institute of Applied Physics NAS of Ukraine, Sumy, Ukraine

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Resonant e^+e^--pair production by an electron in a magnetic field near the process threshold is analytically studied. Using the Nikishov's theorem we estimate the number of events in the magnetic field equivalent to laser wave in the SLAC experiment. The obtained estimate is in reasonable agreement with the experimental data.

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1. INTRODUCTION

Quantum electrodynamic processes in strong magnetic field keep their urgency for both theoretical and experimental study. Strong magnetic field modifies physical processes and allows new ones to occur, like e^+e^- pair production by a moving electron.

The field strength is measured with respect to the so called critical Schwinger field $B_c = m^2c^3/e\hbar \approx 4.4 \cdot 10^{13}$ G. When magnetic field strength B is comparable with the critical one, the quantum electrodynamical treatment of QED processes is necessary.

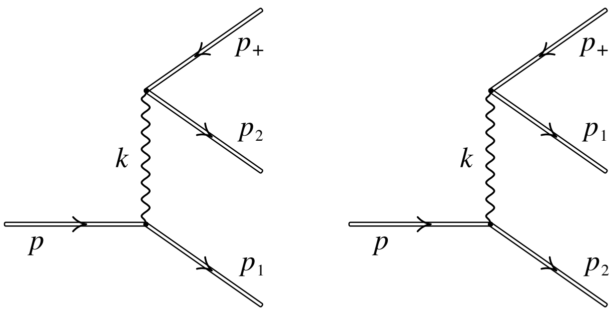
Critical and subcritical magnetic fields are not feasible in laboratory conditions at present time. The strongest field was created using explosive generators [1] and had the strength of about 30 MG, which is still much less than B_c. Nevertheless, neutron stars are believed to have surface magnetic fields within the range from 10^{12} G for radiopulsars to 10^{15} G for magnetars. Thus, QED processes in magnetic field are of great importance in astrophysics.

It is necessary to mention the possibility to study QED processes in subcritical magnetic field in experiments on heavy ion collisions [2]. If the impact parameter has order of magnitude $\sim 10^{-10}$ cm, then magnetic field of moving ions can approach magnitude of $\sim 10^{12}$ G in the region between the ions. These experiments could be carried out at the facilities like LHC or FAIR, which is under construction at the present time in GSI (Darmstadt, Germany).

It is necessary to notice that reaction of e^+e^- pair production by an electron in intense *laser* field was experimentally studied at SLAC [3]. Up to 106 ± 14 events were reported to be observed in collisions of ~ 50 GeV electron beam with terawatt laser pulses. In Ref. [4] this process was studied numerically.

In the present work the second-order process of e^+e^- pair production by an electron in strong magnetic field (see Figure) is analytically studied near the process threshold. Analytical expressions for the total process rate were obtained for subcritical magnetic field strength $B \lesssim B_c$. It is shown, that the main contribution to the rate is determined by the resonant case in agreement with Refs. [3, 4].

*Corresponding author E-mail address: novak-o-p@ukr.net



Feynman diagrams of e^+e^--pair production by an electron in the magnetic field. Double lines represent solutions of Dirac equation for the electron in the magnetic field

2. PROCESS RATE

The process of e^+e^- pair production of electron in magnetic field can be described by two exchange Feynman graphs as on the Figure, where double lines represent solutions of Dirac equation in magnetic field. The calculations have been made in frame of Furry picture, thus the following condition should be satisfied:

$$b = \frac{B}{B_c} \ll 1. \tag{1}$$

The process is considered near the threshold, when the final particles occupy ground Landau levels and the initial electron energy is (hereinafter relativistic units with $\hbar = c = 1$ are used):

$$E = 3m. \tag{2}$$

Thus, final spin states are determined.

Lorentz transformation to the reference frame moving along magnetic field does not change the field, therefore without loss of generality one can set longitudinal momentum of the initial electron to zero

$$p_z = 0.$$

The process probability amplitude reads:

$$S_{fi} = i\alpha \iint d^4x\, d^4x' \times \times \left[(\bar{\Psi}_2 \gamma^\mu \Psi) D_{\mu\nu} (\bar{\Psi}'_1 \gamma^\nu \Psi'_+) - -(\bar{\Psi}_1 \gamma^\mu \Psi) D_{\mu\nu} (\bar{\Psi}'_2 \gamma^\nu \Psi'_+) \right], \quad (3)$$

where $D_{\mu\nu}$ is the photon propagator,

$$D_{\mu\nu} = \frac{g_{\mu\nu}}{(2\pi)^4} \int d^4k\, e^{-ik(x-x')} \frac{4\pi}{k^i k_i}. \quad (4)$$

When process kinematics allows to be $k^2 = 0$, then a resonant divergence arises. In this case the intermediate photon becomes real and the interaction range goes to infinity [5]. The divergence was eliminated introducing small imaginary part into the propagator denominator using Breit-Wigner prescription:

$$\omega \to \omega - i\frac{\Delta}{2}, \quad (5)$$

where Δ is the corresponding state width. Nonresonant contribution to the process is negligible compared to the resonant part.

The general expressions for the process rate near the threshold with account of spin projections is obtained in Ref. [6]. Taking into account, that interference of the diagrams can be neglected, the rate takes on the following form:

$$W^+ \approx \frac{\alpha^2 m}{3\pi^2 \sqrt{3} l!} Y, \quad (6)$$

$$W^- \sim b W^+, \quad (7)$$

where superscript denotes the initial electron spin projection, α is fine structure constant, l is Landau level number of the initial electron, $b = B/B_c$, B is the magnetic field, B_c is the critical field, and

$$Y = \iint ds\, du \left| e^{-s^2} D \right|^2, \quad (8)$$

$$D = \int \frac{(s + iq)^l}{r^2 - q^2} e^{-q^2 - 2iuq} dq. \quad (9)$$

Here, the following notations are used:

$$\begin{aligned} s &= m\Omega(x_0 - x_{01}), \\ u &= m\Omega(x_0 - x_{02}), \\ q &= k_x / m\sqrt{2b}, \\ r^2 &= \Omega^2 - s^2, \qquad \Omega^2 = 2/b, \end{aligned} \quad (10)$$

x_0, x_{01} and x_{02} are x-coordinates of classical orbit centers of initial and final electrons respectively, k_x is x-component of intermediate photon momentum.

The analysis shows, that the main contribution in the integral (9) is made by the summand with s^l when $r^2 > 0$ and the integrand has a singularity.

Taking into account, that the quantity Δ is small, integration in Eqs. (9)-(8) can be carried out analytically. After performing corresponding calculations it could be found that

$$Y = b\pi^2 \sqrt{\pi} \frac{\Omega^{2l} e^{-2\Omega^2}}{\Delta/m} \frac{\Gamma(l + 1/2)}{l!}, \quad (11)$$

where $\Gamma(l + 1/2)$ is gamma-function.

Averaging the rate over the initial electron spin projection, finally we obtain (in CGS units):

$$W = \alpha^2 \left(\frac{mc^2}{\hbar} \right) \frac{b\sqrt{\pi}}{6\sqrt{3}} \frac{\Omega^{2l} e^{-2\Omega^2}}{\Delta/m} \frac{\Gamma(l + 1/2)}{(l!)^2}. \quad (12)$$

In the resonant case, the total probability can be expressed via the product of the rates of the first-order processes of photon radiation and pair production by a single photon. Near the process threshold, when the condition $E \approx 3m$ is true and consequently $bl = 4$ and $l \gg 1$ are also valid, the process rate can be found in the following form:

$$W = \frac{\sqrt{b}}{3\sqrt{6}} \frac{W_{e\to\gamma e} W_{\gamma\to ee^+}}{\Delta}. \quad (13)$$

Here, $W_{e\to\gamma e}$ and $W_{\gamma\to ee^+}$ are the rates of the corresponding first-order processes [7, 8]:

$$W_{e\to\gamma e} = \alpha m \sqrt{\pi} \frac{\Omega^{2l} e^{-\Omega^2}}{\Gamma(l + 1/2) l}, \quad (14)$$

$$W_{\gamma\to ee^+} = \alpha m \frac{b e^{-\Omega^2}}{\sqrt{2\, \delta E/m}}, \quad (15)$$

where $\delta E = E - 3m \sim mb$ is the kinematic factor.

The intermediate photon width is determined mainly by the total radiation rate of the initial electron. The radiation process was studied in a number of works, e. g. [8–13].

As an example, let us calculate the rate (12) when field strength is $b = 0.1$ ($B \approx 4.4 \cdot 10^{12}$ G):

$$\Delta \approx 7 \cdot 10^{17} \text{s}^{-1}, \quad (16)$$

$$W = 7 \cdot 10^3 \text{ s}^{-1}. \quad (17)$$

3. DISCUSSION

It should be noted, that according to Lorentz transformation arbitrary electromagnetic field goes to almost equal crossed electric and magnetic fields in the rest frame of a relativistic particle. This means physical equivalence of any field configuration and electromagnetic wave, if the field changes slowly in comparison with characteristic electromagnetic time ($\sim 10^{-21}$ s). The latter statement known as Nikishov theorem [14] allows to compare the result (12) with SLAC experiment on observation of e^+e^- pair production by electron in intense laser wave.

For this purpose one should calculate the rate (12) in the equivalent magnetic field B_{eq} and carry out averaging over the wave oscillations [14]:

$$W_{emw} = \frac{2}{\pi} \int_0^{\pi/2} W(B_{eq} \sin\phi) d\phi, \tag{18}$$

where $B_{eq} = 2F_{emw}$ and F_{emw} is the strength of an electromagnetic wave.

However, Eq. (12) is true near the process threshold only, when the condition $E \approx 3m$ is fulfilled. Therefore before the comparison with the SLAC experiment it is necessary to pass from the laboratory frame to the "threshold" one where the electron beam energy is $E \approx 3m$. The amplitude value of equal magnetic field in the threshold frame is $B_{eq} \approx 6.1 \cdot 10^{12}$ G, and $b \approx 0.14$ respectively.

When calculating the rate Eq. (12) it is necessary to take into account limited interaction time as well as the radiative width (16). Therefore the intermediate state width is a sum of the radiative width and the quantity $1/\Delta t_T$, where Δt_T is laser-electron interaction time in the threshold frame.

The number of produced pairs is

$$N_{e^+e^-} = k \cdot N_{int}(1 - e^{-W_{eq}\Delta t_T}), \tag{19}$$

where $k = 21\ 962$ is number of collisions of the electron and laser beams [3], N_{int} is the number of electrons in the interaction region. The values of N_{int} and Δt_T can be estimated using the data from Ref. [3]: $\Delta t_L \approx 0.002$ fs, $N_{int} \sim 1.7 \cdot 10^8$. Finally, the number of events according to Eq. (19) is

$$N_{e^+e^-} \approx 80, \tag{20}$$

which is in reasonable agreement with the experimental result of 106 ± 14 indicated in Ref. [3].

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ПОРОГОВОЕ РОЖДЕНИЕ ЭЛЕКТРОН-ПОЗИТРОННОЙ ПАРЫ ПОЛЯРИЗОВАННЫМ ЭЛЕКТРОНОМ В СИЛЬНОМ МАГНИТНОМ ПОЛЕ

А.П. Новак, Р.И. Холодов

Аналитически исследовано образование e^+e^-–пары электроном в магнитном поле вблизи порога процесса. Произведена оценка числа событий в эксперименте SLAC с использованием теоремы Никишова. Полученное число событий согласуется с экспериментальными данными.

ПОРОГОВЕ НАРОДЖЕННЯ ЕЛЕКТРОН-ПОЗИТРОННОЇ ПАРИ ПОЛЯРИЗОВАНИМ ЕЛЕКТРОНОМ В СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

О.П. Новак, Р.І. Холодов

Аналітично досліджено утворення e^+e^-–пари електроном в магнітному полі поблизу порогу процесу. Одержана оцінка числа подій в експерименті SLAC з використанням теореми Нікішова. Знайдене число подій узгоджується з експериментальними даними.