INCOHERENT BREMSSTRAHLUNG IN FLAT AND BENT CRYSTALS

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Incoherent bremsstrahlung by high-energy particles in crystal is due to the thermal spread of atoms in relation to their equilibrium positions in the lattice. The simulation procedure developed earlier for the incoherent radiation is applied to the case of the electrons and positrons motion in the sinusoidally bent crystal. The results of simulation are in agreement with the data of recent experiments carried out at the Mainz Microtron MAMI. The possibility of use of the sinusoidally bent crystals as undulators is discussed.

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1. INTRODUCTION

The bremsstrahlung cross section for relativistic electrons in a crystal is split into the sum of the coherent part (due to the spatial periodicity of the atoms' arrangement in the crystal) and the incoherent one (due to the thermal motion of atoms in the crystal) [1,2]. Although the spectrum of incoherent radiation in crystal is similar to one in amorphous medium, the incoherent radiation intensity could demonstrate substantial dependence on the crystal orientation due to the electrons' flux redistribution in the crystal (channeling etc.). The simulation based on the semiclassical description of the radiation process [3–5] confirms that viewpoint. The results of simulation are in a good agreement with the corresponding early [6] and recent [7] experimental data.

Here we present the results of simulation using the improved procedure taking into account the crystal deformations. The simulation was carried out under the conditions of the recent experiment performed at the Mainz Microtron MAMI [8] to explore the radiation emission from periodically bent crystal. The possibility of application of such crystals as undulators is discussed during last years [8–10].

2. COMPUTATION METHOD

Let us consider the high energy electron incidence on the atomic string in the crystal. The two-dimensional multiple scattering angle $\boldsymbol{\vartheta}$ is equal to the sum of individual scattering angles on the atoms:

$$\boldsymbol{\vartheta} = \sum_n \boldsymbol{\vartheta}(\boldsymbol{\rho}_n), \qquad (1)$$

where $\boldsymbol{\rho}_n$ is the impact parameter of the collision with the n-th atom of the crystal. The mean square of the multiple scattering angle (averaged over the thermal vibrations of atoms) can be expressed as the sum of two blocks of terms:

$$\begin{aligned}\left\langle \left| \sum_n \boldsymbol{\vartheta}(\boldsymbol{\rho}_n) \right|^2 \right\rangle &= \sum_{n,m} \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n) \rangle \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_m) \rangle \\ &+ \sum_n \left\{ \left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n)^2 \right\rangle - \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n) \rangle^2 \right\} \\ &\equiv \left\langle \boldsymbol{\vartheta}^2 \right\rangle_{\text{coh}} + \left\langle \boldsymbol{\vartheta}^2 \right\rangle_{\text{incoh}} . \end{aligned} \qquad (2)$$

The first block describes the coherent scattering which can be interpreted as a motion in the uniform string potential [2,11]. The second one describes the incoherent scattering of the electron on the thermal vibrations of the lattice atoms.

The bremsstrahlung spectrum of the electrons passing through the crystal also can be expressed as the sum of the coherent and incoherent contributions [1,2]. For the electrons of the energy $\varepsilon \sim 1$ GeV the main contribution of the coherent effect is made to the soft range of the spectrum (the photon energy $\hbar\omega$ is less or of the order of dozens of MeVs). In the medium and hard ranges of the spectrum the incoherent radiation is predominant.

The spectral density of the incoherent radiation from the individual electron moving on the given trajectory is described by the formula [4,5]

$$\begin{aligned}\left(\frac{d\mathcal{E}}{d\omega} \right)_{\text{incoh}} &= \frac{2e^2 \varepsilon (\varepsilon - \hbar\omega)}{3\pi m^2 c^5} \left\{ 1 + \frac{3}{4} \frac{(\hbar\omega)^2}{\varepsilon(\varepsilon - \hbar\omega)} \right\} \\ &\times \sum_n \left\{ \left\langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n)^2 \right\rangle - \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n) \rangle^2 \right\}, \end{aligned} \qquad (3)$$

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where m and e are the electron's mass and charge, c is the velocity of light. It is convenient to compare the incoherent radiation intensity of the uniform electron beam in the crystal to the corresponding intensity in the amorphous medium (described by Bethe-Heitler formula). The ratio of these two values is equal to [4,5]

$$N_\gamma = \frac{1}{2}\pi N R^2 \ln(mRc/\hbar) \times \int d^2\rho_0 \sum_n \left\{ \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n)^2 \rangle - \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}_n) \rangle^2 \right\}, \quad (4)$$

where N is the total number of the electron's collisions with atoms under its motion through the crystal, R is Thomas-Fermi radius of the atom, integration over $d^2\rho_0$ means the integration over all possible points of incidence of the electron on the crystal in the limits of one elementary cell.

The values of the function $F(\rho) = \langle \boldsymbol{\vartheta}(\boldsymbol{\rho})^2 \rangle - \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}) \rangle^2$ are determined by linear interpolation of the values pre-calculated on the regular grid of impact parameters [5]. The impact parameters $\boldsymbol{\rho}_n$ are found using the electron's trajectory obtained by numerical integration of the equation of motion in the field of the set of parallel uniform strings. The influence of the incoherent scattering by the thermal vibrations of atoms on the electron's trajectory can be taken into account by adding to each component of the electron's transverse velocity the random value with the dispersion $c\sqrt{\left\{ \langle \boldsymbol{\vartheta}(\boldsymbol{\rho})^2 \rangle - \langle \boldsymbol{\vartheta}(\boldsymbol{\rho}) \rangle^2 \right\}/2}$ after each collision. For the further computational details see [4,5].

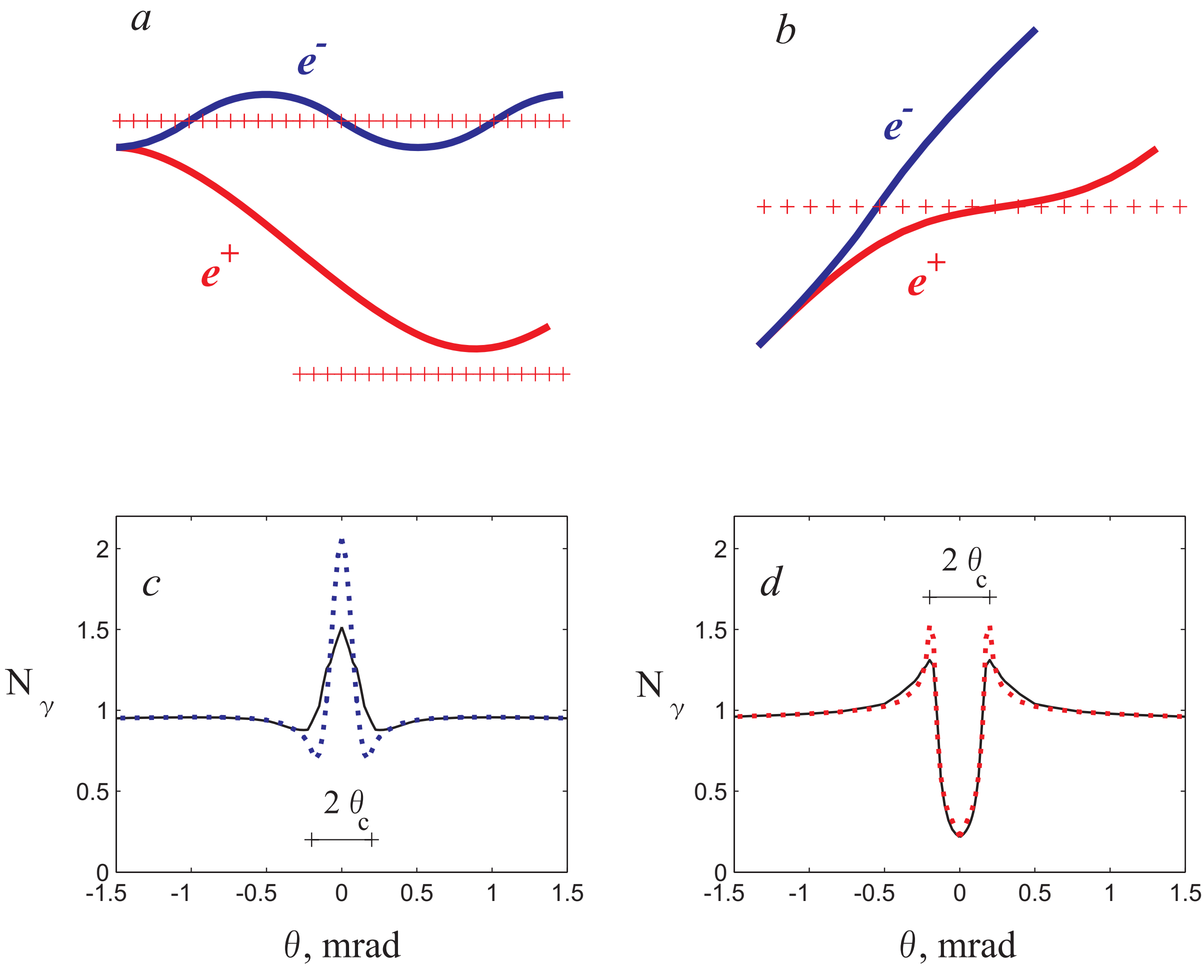

Fig. 1. **(a)** *Typical trajectories of the electrons and positrons under planar channeling. Pluses mark the positions of atomic strings (perpendicular to the figure plane) forming the atomic planes of the crystal. The horizontal scale of the figure is highly compressed.* **(b)** *The same for above-barrier motion.* **(c)** *Simulated incoherent bremsstrahlung intensity (in ratio to the Bethe-Heitler intensity in amorphous medium, see Eq. (4)) from 1 GeV electrons vs incidence angle θ to (110) plane of 30 μm thick Si crystal [3]. Dotted line corresponds to the trajectories simulated neglecting thermal vibrations of atoms.* **(d)** *The same for positrons*

3. RESULTS OF SIMULATION

The origin of the orientation dependence of the incoherent radiation intensity is illustrated in Fig. 1. When the electron is incident to the atomic plane of the crystal under angle θ less than some critical angle θ_c, it can be captured by the attractive potential of the plane. The finite motion in that potential is called as the planar channeling (see, e.g., [2,11]).

Under planar channeling the electrons collide with atoms under small impact parameters more frequently than in amorphous medium, that leads to the increase of the incoherent bremsstrahlung intensity; for above-barrier motion the situation is opposite. The account of the incoherent scattering of the particles on the thermal vibrations of the atoms leads to dechanneling and, hence, to the smoothing of the describer orientation dependence (compare solid and dotted lines in Fig. 1, (c) and 1, (d)).

In the present article the simulation was carried out under the conditions of the experiment [8], where the radiation from $\varepsilon = 855$ MeV electrons under their incidence onto the silicon crystal with sinusoidally bent (110) planes had been studied. The yield of photons with the energy $\hbar\omega = \varepsilon/2$ (for which the incoherent mechanism of bremsstrahlung is predominant) had been registered.

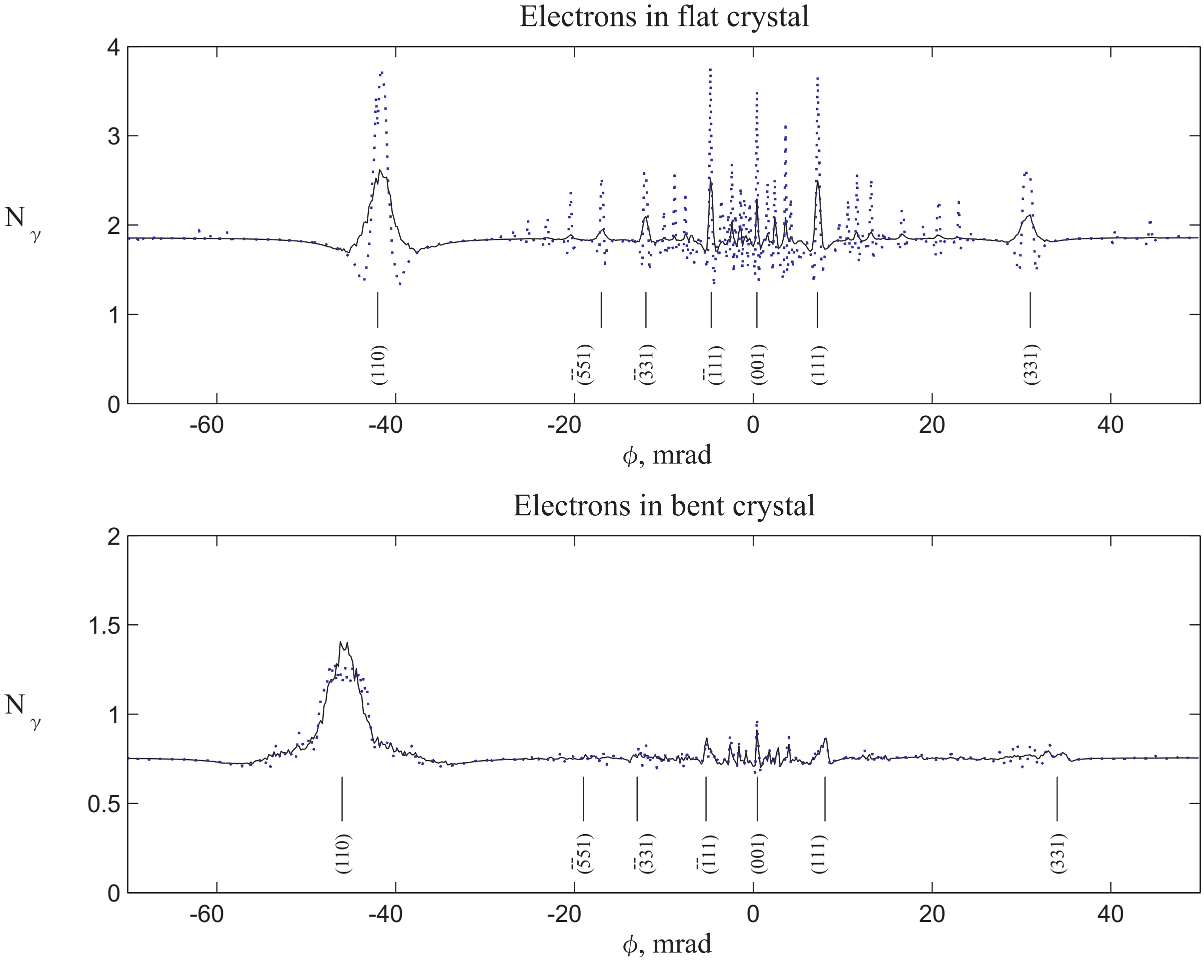

Fig. 2. *The incoherent bremsstrahlung intensity (in ratio to Bethe-Heitler intensity in amorphous medium) from 855 MeV electrons in flat (upper plot) and sinusoidally bent (lower plot) silicon crystals under scanning of the goniometric angle ϕ like in the experiment [8]*

The results of simulation (Fig. 2) demonstrate the qualitative agreement with the experimental data [8]. We can see characteristic structures similar to one in Fig. 1, (c), generated by different crystallographic planes with the common $[1\bar{1}0]$ axis.

The decrease of the radiation intensity in comparison to the reference flat crystal permits to estimate the reduction of the dechanneling length due to the crystal bending. We can see that the bending of the crystallographic planes increases the dechanneling rate so highly that the scattering on the thermal vibrations of atoms already have no substantial influence on the incoherent radiation intensity (compare solid and dashed curves on the lower panel of Fig. 2).

The main cause of the rapid dechanneling in the bent crystal lies in the arising of the centrifugal addition to the planar potential [11]:

$$U_{eff} = U(x) - \varepsilon\frac{x}{R_b} \quad \text{under} \quad |x| \gg R_b \qquad (5)$$

(where R_b is the bending radius) and, as a consequence, to the elimination of the potential barriers between which the channeling could take the place. In our case the height of the potential barrier in the flat crystal is about $U_{max} \approx 23.5$ eV, and the centrifugal correction on the half of the interplanar distance $\Delta U_c(\Delta x = d_p/2) \approx 13.2$ eV for the maximal bending of the sinusoid, that corresponds to the curve 2 in Fig. 3. So, only a small part of incident particles could be captured into the channeling regime.

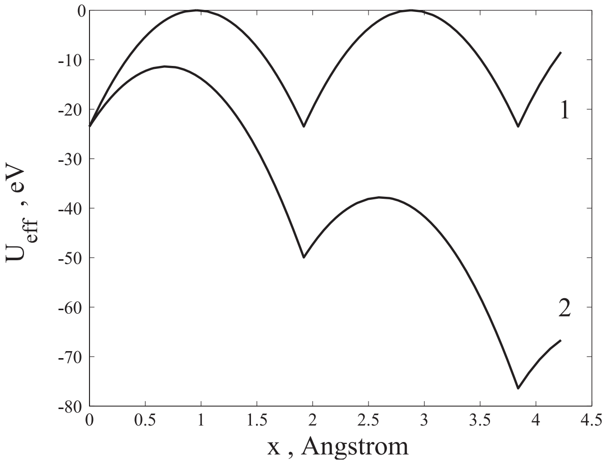

Fig. 3. *Potential energy of the electron in the planar potentials of the flat (curve 1) and bent (curve 2) crystals*

4. CONCLUSIONS

The results of simulation demonstrate that the planar channeling effect leads to the orientation dependence of the incoherent bremsstrahlung intensity not only on flat, but also in bent crystals. However, in the bent crystal the electrons rapidly leave the planar channels. That leads to doubt about the efficiency of sinusoidally bent crystals as undulators. As we can see from Fig. 4, the use of the positron beam instead of the electron one could not seriously improve the situation.

Qualitatively similar results could be expected for the nuclear reactions yield in bent crystals [12].

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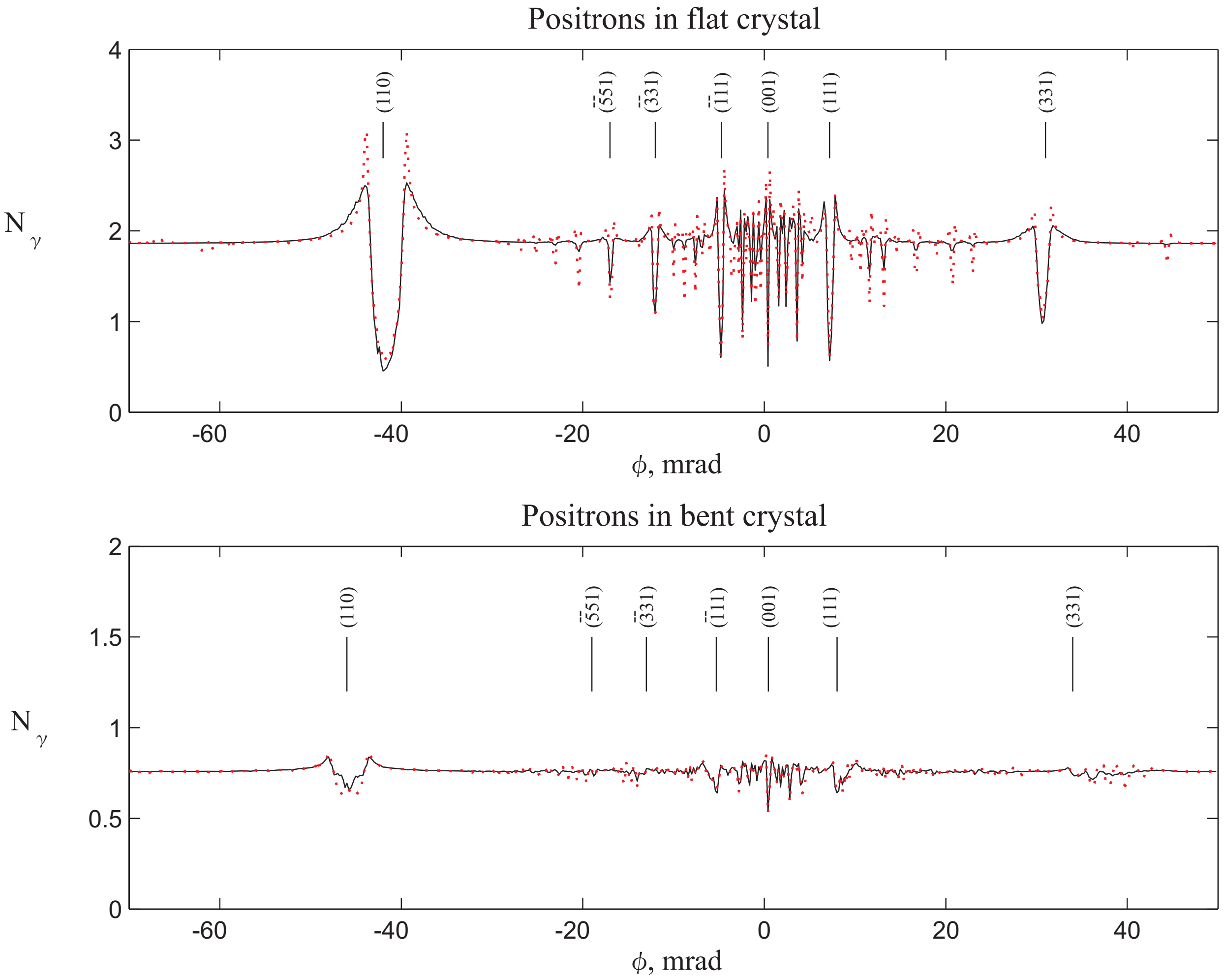

Fig. 4. *The same as in Fig. 2 for positrons*

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НЕКОГЕРЕНТНОЕ ТОРМОЗНОЕ ИЗЛУЧЕНИЕ В ПРЯМЫХ И ИЗОГНУТЫХ КРИСТАЛЛАХ

Н.Ф. Шульга, В.В. Сыщенко, А.И. Тарновский

Некогерентное тормозное излучение частиц высокой энергии в кристалле обусловлено тепловым разбросом атомов относительно их равновесных положений в решетке. Развитая ранее процедура моделирования некогерентного излучения применена к случаю движения электронов и позитронов в синусоидально изогнутом кристалле. Результаты моделирования согласуются с данными недавних экспериментов на микротроне MAMI в Майнце. Обсуждается перспективность использования синусоидально изогнутых кристаллов в качестве ондуляторов.

НЕКОГЕРЕНТНЕ ГАЛЬМІВНЕ ВИПРОМІНЮВАННЯ У ПРЯМИХ ТА ВИГНУТИХ КРИСТАЛАХ

М.Ф. Шульга, В.В. Сищенко, А.І. Тарновський

Некогерентне гальмівне випромінювання частинок високої енергії у кристалі обумовлено тепловим розкидом атомів відносно їх рівноважних положень у решітці. Процедура моделювання некогерентного випромінювання, що розвита раніше, пристосовується до випадку руху електронів та позитронів у сінусоїдально вигнутому кристалі. Результати моделювання знаходяться у відповідності до даних недавніх експериментів на мікротроні MAMI у Майнці. Обговорюється перспективність використання сінусоїдально вигнутих кристалів як ондуляторів.