A NONLOCAL EXTENSION OF THE WENTZEL FIELD MODEL IN THE CLOTHED-PARTICLE REPRESENTATION

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The clothed particle approach is applied to express the total Hamiltonian of interacting fields in terms of clothed particles. In order to avoid ultraviolet divergences typical of many field theories we introduce some covariant cutoff functions in momentum space in the Wentzel field model. We will show how in the framework of the nonlocal meson-boson field model one can build interactions between the clothed mesons and bosons. Moreover, the mass renormalization terms, that are compulsory to ensure the relativistic invariance of the theory as a whole (in Dirac's sense), turn out to be expressed through certain covariant integrals. They are convergent in the field model with appropriate cutoff factors.

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1. INTRODUCTION

Following our recent work [1] we will show how an algebraic approach proposed there for constructing the generators of the Poincaré group can be realized within a nonlocal extension of the so-called Wentzel model. Our departure point is a nonlocal Hamiltonian for interacting fields, that can be built up by introducing some "cutoff" function (shortly the g-factor) in every vertex which is associated with particle creation and/or annihilation. As usually, such g-factors are needed, first of all, to carry out finite intermediate calculations trying to remove ultraviolet divergences inherent in local field models. However, in the instant form of relativistic dynamics used here it is very important to take into account certain constraints imposed upon such cutoffs to meet requirements of special relativity and other symmetries, e.g., with respect to charge conjugation, space inversion and time reversal.

We have managed to do it [1] by defining a covariant generating function for the cutoffs in case of trilinear Yukawa-type couplings. The function, being dependent on some Lorentz scalars composed of the particle three-momenta, plays a central role when integrating the Poincaré commutators to derive then the clothed-particle representation (CPR) expressions for the Hamiltonian, the boost operators, the mass renormalization terms and so on accordingly [2].

Moreover, it is expected that by choosing the g-factors in a proper way (for instance, as square integrable functions of particle momenta) one can get rid of certain drawback of field models with local interactions (see [1]).

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2. METHOD OF UNITARY CLOTHING TRANSFORMATIONS

As before (see, e.g., [3]), let us remind that the UCT method exposed in [1–4] is aimed to express a given field Hamiltonian

$$\begin{aligned} H \equiv H(\alpha) &= H_F(\alpha) + H_I(\alpha) \\ &= W(\alpha_c)H(\alpha_c)W^\dagger(\alpha_c) \equiv K(\alpha_c), \end{aligned} \tag{1}$$

primarily dependent on the α set of "bare" particle creation and annihilation operators, through their "clothed" counterparts α_c via the unitary transformation W. The latter removes from the interaction $V(\alpha)$ that enters $H_I(\alpha) = V(\alpha) + V_{ren}(\alpha)$ the so-called "bad" terms. By definition, such terms prevent the physical vacuum $|\Omega\rangle$ (the H lowest eigenstate) and the one-clothed-particle states $|n\rangle_c = a_c^\dagger(n)|\Omega\rangle$ to be the H eigenvectors for all n included. The bad terms occur every time when any normally ordered product

$$a^\dagger(1')a^\dagger(2')...a^\dagger(n'_C)a(n_A)...a(2)a(1)$$

of the class [C.A] embodies, at least, one substructure which belongs to one of the classes $[k.0]$ $(k = 1, 2, ...)$ and $[k.1]$ $(k = 0, 1, ...)$. Our consideration is focused upon various field models (local and nonlocal) in which the interaction density $H_I(\mathbf{x})$ consists of scalar $H_{sc}(\mathbf{x})$ and nonscalar $H_{nsc}(\mathbf{x})$ contributions:

$$H_I(\mathbf{x}) = H_{sc}(\mathbf{x}) + H_{nsc}(\mathbf{x}), \tag{2}$$

where the property to be a scalar means

$$U_F(\Lambda)H_{sc}(x)U_F^{-1} = H_{sc}(\Lambda x), \quad \forall x = (t, \mathbf{x}) \tag{3}$$

for all Lorentz transformations Λ.

Therefore, we have

$$H_I(\alpha) = \int H_I(\mathbf{x})d\mathbf{x} = H_{sc}(\alpha) + H_{nsc}(\alpha), \qquad (4)$$

$$H_{sc(nsc)}(\alpha) = \int H_{sc(nsc)}(\mathbf{x})d\mathbf{x},$$

$$H_{sc}(\alpha) = V_{bad}(\alpha) + V_{good}(\alpha)$$

to eliminate the bad part V_{bad} from the similarity transformation

$$\begin{aligned} K(\alpha_c) &= W(\alpha_c)[H_F(\alpha_c) + H_I(\alpha_c)]W^\dagger(\alpha_c) \\ &= W(\alpha_c)[H_F(\alpha_c) + V_{bad}(\alpha_c) \\ &+ V_{good}(\alpha_c) + H_{nsc}(\alpha_c)]W^\dagger(\alpha_c). \end{aligned} \qquad (5)$$

For the unitary clothing transformation (UCT) $W = \exp R$ with $R = -R^\dagger$ it is implied that we will eliminate the bad terms V_{bad} in the r.h.s. of

$$\begin{aligned} K(\alpha_c) &= H_F(\alpha_c) + V_{bad}(\alpha_c) + [R, H_F] \\ &+ [R, V_{bad}] + \frac{1}{2}[R,[R,H_F]] \\ &+ \frac{1}{2}[R,[R,V_{bad}]] + ... + e^R V_{good} e^{-R} + e^R H_{nsc} e^{-R} \end{aligned} \qquad (6)$$

by requiring that

$$[H_F, R] = V_{bad} \qquad (7)$$

for the operator R of interest.

One should note that unlike the original clothing procedure we eliminate here the bad terms only from H_{sc} interaction in spite of such terms can appear in the nonscalar interaction as well (details in [5]).

Now, we get the division

$$H = K(\alpha_c) = K_F + K_I \qquad (8)$$

with a new free part $K_F = H_F(\alpha_c) \sim a_c^\dagger a_c$ and interaction

$$\begin{aligned} K_I &= V_{good}(\alpha_c) + H_{nsc}(\alpha_c) + [R, V_{good}] \\ &+ \frac{1}{2}[R, V_{bad}] + [R, H_{nsc}] + \frac{1}{3}[R,[R,V_{bad}]] + ..., \end{aligned} \qquad (9)$$

where the r.h.s. involves along with good terms other bad terms to be removed via subsequent UCTs.

3. A NONLOCAL EXTENSION OF THE WENTZEL FIELD MODEL

As an illustration, let us consider the field model of "scalar nucleons" (more precisely, charged spinless bosons) and neutral scalar bosons, in which

$$H_I = V_{nloc} + M_s + M_b \qquad (10)$$

with the normally ordered interaction

$$\begin{aligned} V_{nloc} &= \frac{1}{2[2(2\pi)^3]^{1/2}} \int \frac{d\mathbf{p}'}{E_{\mathbf{p}'}} \int \frac{d\mathbf{p}}{E_{\mathbf{p}}} \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} \\ &\times \{\delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) g_{11}(p', p, k) b^\dagger(p') b(p) a(k) \\ &+ \delta(\mathbf{p}' + \mathbf{p} - \mathbf{k}) g_{12}(p', p, k) b^\dagger(p') d^\dagger(p) a(k) \\ &+ \delta(\mathbf{p}' + \mathbf{p} + \mathbf{k}) g_{21}(p', p, k) d(p') b(p) a(k) \\ &+ \delta(\mathbf{p}' - \mathbf{p} - \mathbf{k}) g_{22}(p', p, k) d^\dagger(p') d(p) a(k)\} + \text{H.c.} \end{aligned} \qquad (11)$$

Adopting the convention

$$\begin{aligned} &[b^\dagger(p'), d(p')] \begin{bmatrix} X_{11}(p',p) & X_{12}(p',p) \\ X_{21}(p',p) & X_{22}(p',p) \end{bmatrix} \begin{bmatrix} b(p) \\ d^\dagger(p) \end{bmatrix} \\ &= F^\dagger_{\varepsilon'}(p') X_{\varepsilon'\varepsilon}(p',p) F_\varepsilon(p) \equiv F^\dagger_b(p') X(p',p) F_b(p) \end{aligned} \qquad (12)$$

we can write in more compact form

$$V_{nloc} = V_b + V_b^\dagger, \;\; V_b = \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} : F_b^\dagger G(k) F_b : a(k).$$

Matrix $G(k)$ is composed of elements

$$\begin{aligned} G_{\varepsilon'\varepsilon}(p',p,k) &= \frac{1}{2[2(2\pi)^3]^{1/2}} \bar{g}_{\varepsilon'\varepsilon}(p',p,k) \\ &\times \delta(\mathbf{k} - (-1)^\varepsilon \mathbf{p} + (-1)^{\varepsilon'} \mathbf{p}'), \quad (\varepsilon', \varepsilon = 1, 2), \end{aligned} \qquad (13)$$

where $\bar{g}_{\varepsilon'\varepsilon}(p',p,k)$ coincide with $g_{\varepsilon'\varepsilon}(p',p,k)$ except $\bar{g}_{22}(p',p,k) = g_{22}(p,p',k)$.

It is implied that operators $a(a^\dagger)$, $b(b^\dagger)$ and $d(d^\dagger)$ meet commutation relations

$$[a(k), a^\dagger(k')] = k_0 \delta(\mathbf{k} - \mathbf{k}'), \qquad (14)$$

$$[b(p), b^\dagger(p')] = [d(p), d^\dagger(p')] = p_0 \delta(\mathbf{p} - \mathbf{p}'), \qquad (15)$$

with all the remaining ones being zero. Here $k_0 = \omega_{\mathbf{k}} = \sqrt{\mathbf{k}^2 + \mu_s^2}$ ($p_0 = E_{\mathbf{p}} = \sqrt{\mathbf{p}^2 + \mu_b^2}$) is the energy of the neutral (charged) particle with mass $\mu_s(\mu_b)$. For our nonlocal model we will retain the property to be Lorentz scalar assuming

$$U_F(\Lambda) V_{nloc}(x) U_F^{-1}(\Lambda) = V_{nloc}(\Lambda x). \qquad (16)$$

It is readily seen that this relation holds if the coefficients $g_{\varepsilon'\varepsilon}$ meet the condition

$$g_{\varepsilon'\varepsilon}(\Lambda p', \Lambda p, \Lambda k) = g_{\varepsilon'\varepsilon}(p', p, k). \qquad (17)$$

On the mass shell with $p'^2 = p^2 = \mu_b^2$ and $k^2 = \mu_s^2$ the latter means that functions $g_{\varepsilon'\varepsilon}(p',p,k)$ can depend only upon invariants $p'p$, $p'k$, pk.

These cutoffs are subject to other constraints imposed by different symmetries. For example, invariance of the hermitian operator V_{nloc} with respect to: i) space inversion; ii) time reversal and iii) charge conjugation yields the relations:

$$g_{\varepsilon'\varepsilon}(p',p,k) = g_{\varepsilon'\varepsilon}(p,p',k), \quad \varepsilon' \neq \varepsilon \qquad (18)$$

$$g_{\varepsilon'\varepsilon}(p',p,k) = g_{\varepsilon'\varepsilon}(p'_-, p_-, k_-), \qquad (19)$$

$$g_{11}(p',p,k) = g_{22}(p',p,k). \qquad (20)$$

"Mass renormalization" terms M_s and M_b can be represented in the form:

$$\begin{aligned} M_s &= \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}^2} \{m_1(k) a^\dagger(k) a(k) \\ &+ m_2(k)[a^\dagger(k) a^\dagger(k_-) + a(k) a(k_-)]\} \end{aligned} \qquad (21)$$

and

$$\begin{aligned} M_b &= \int \frac{d\mathbf{p}}{E_{\mathbf{p}}^2} \{m_{11}(p) b^\dagger(p) b(p) + m_{12}(p) b^\dagger(p) d^\dagger(p_-) \\ &+ m_{21}(p) b(p) d(p_-) + m_{22}(p) d^\dagger(p) d(p)\}, \end{aligned} \qquad (22)$$

where the coefficients $m_{1,2}(k)$ and $m_{\varepsilon'\varepsilon}(p',p)$, being for the time unknown, may be momentum dependent.

4. GENERATORS FOR CLOTHED PARTICLES. ELIMINATION OF BAD TERMS

At this point we will come back to our model with $V_{bad} = V_{nloc}$, $V_{good} = 0$ and $R = R_{nloc}$ to calculate the simplest commutator $[R_{nloc}, V_{nloc}]$ in which the clothing operator R_{nloc} is determined by

$$[H_F, R_{nloc}] = V_{nloc}. \tag{23}$$

From the equation it follows that its solution can be given by

$$R_{nloc} = \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} : F_b^{\dagger} R(k) F_b : a(k) - \text{H.c.}$$
$$= \mathcal{R}_{nloc} - \mathcal{R}_{nloc}^{\dagger}. \tag{24}$$

The matrix $R(k)$ is composed of the elements:

$$R_{\varepsilon'\varepsilon}(p', p, k) = -\frac{\bar{g}_{\varepsilon'\varepsilon}(p', p, k)}{\omega_{\mathbf{k}} + (-1)^{\varepsilon'} E_{\mathbf{p}'} - (-1)^{\varepsilon} E_{\mathbf{p}}}$$
$$\times \delta(\mathbf{k} + (-1)^{\varepsilon'} \mathbf{p}' - (-1)^{\varepsilon} \mathbf{p}) \quad (\varepsilon', \varepsilon = 1, 2). \tag{25}$$

Such a solution is valid if $\mu_s < 2\mu_b$. In other words, under such an inequality the operator R_{nloc} has the same structure as V_{nloc} itself. After the normal ordering of meson and boson operators in commutator $[R_{nloc}, V_{nloc}]$ one can obtain the $2 \to 2$ interactions of the type $b^{\dagger}a^{\dagger}ba$, $d^{\dagger}a^{\dagger}da$, $b^{\dagger}d^{\dagger}aa$, $a^{\dagger}a^{\dagger}bd$ and $b^{\dagger}b^{\dagger}bb$, $b^{\dagger}d^{\dagger}bd$, $d^{\dagger}d^{\dagger}dd$.

For example, the boson-boson interaction operator can be represented as

$$\frac{1}{2}[R_{nloc}, V_{nloc}](bb \to bb)$$
$$= -\frac{1}{4} \int \frac{d\mathbf{p}_2'}{E_{\mathbf{p}_2'}} \int \frac{d\mathbf{p}_2}{E_{\mathbf{p}_2}} \int \frac{d\mathbf{p}_1'}{E_{\mathbf{p}_1'}} \int \frac{d\mathbf{p}_1}{E_{\mathbf{p}_1}}$$
$$\times \delta(\mathbf{p}_1' + \mathbf{p}_2' - \mathbf{p}_1 - \mathbf{p}_2)$$
$$\times g_{11}(p_1', p_1, k) g_{11}(p_2', p_2, k)$$
$$\times \left\{ \frac{1}{(p_1 - p_1')^2 - \mu_s^2} + \frac{1}{(p_2 - p_2')^2 - \mu_s^2} \right\}$$
$$\times b_c^{\dagger}(p_2') b_c^{\dagger}(p_1') b_c(p_2) b_c(p_1) \tag{26}$$

with $\mathbf{k} = \mathbf{p}_1' - \mathbf{p}_1$. In these equations we meet a covariant (Feynman-like) "propagator"

$$\frac{1}{2} \left\{ \frac{1}{(p_1 - p_1')^2 - \mu_s^2} + \frac{1}{(p_2 - p_2')^2 - \mu_s^2} \right\}, \tag{27}$$

which on the energy shell

$$E_{\mathbf{p}_1} + E_{\mathbf{p}_1} = E_{\mathbf{p}_1'} + E_{\mathbf{p}_2'} \tag{28}$$

is converted into the genuine Feynman propagator for the corresponding S matrix.

5. MASS RENORMALIZATION AND RELATIVISTIC INVARIANCE

We have seen how in the framework of the nonlocal meson-boson model one can build the $2 \to 2$ interactions between the clothed mesons and bosons. They appear in a natural way from the commutator $\frac{1}{2}[R_{nloc}, V_{nloc}]$ as the operators $b^{\dagger}a^{\dagger}ba$, $d^{\dagger}a^{\dagger}da$, $b^{\dagger}b^{\dagger}bb$, $b^{\dagger}d^{\dagger}bd$, $d^{\dagger}d^{\dagger}dd$, $b^{\dagger}d^{\dagger}aa$, $a^{\dagger}a^{\dagger}bd$ of the class [2.2]. Moreover, this commutator is a spring of the good operators $a^{\dagger}a$, $b^{\dagger}b$ and $d^{\dagger}d$ of the class [1.1] together with the bad operators aa and bd of the class [0.2] and their hermitian conjugates $a^{\dagger}a^{\dagger}$ and $b^{\dagger}d^{\dagger}$ of the class [2.0]. These operators may be cancelled by the respective counterterms from

$$H_{nsc}(\alpha) = M_s(\alpha) + M_b(\alpha). \tag{29}$$

Let us show that such a cancellation gives rise to certain definitions of the mass coefficients. Indeed, one can show that

$$\frac{1}{2}[R_{nloc}, V_{nloc}](a^{\dagger}a)$$
$$= -\frac{1}{2} \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}^2} \int \frac{d\mathbf{p}}{E_{\mathbf{p}} E_{\mathbf{p}-\mathbf{k}}} [\frac{g_{21}^2(p, q_-, k_-)}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}}$$
$$+ \frac{g_{12}^2(p, q_-, k)}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}}] a^{\dagger}(k) a(k), \tag{30}$$

where $q = (E_{\mathbf{p}-\mathbf{k}}, \mathbf{p} - \mathbf{k})$. In the same way we obtain

$$\frac{1}{2}[R_{nloc}, V_{nloc}](aa)$$
$$= \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}^2} \int \frac{d\mathbf{p}}{E_{\mathbf{p}}} g_{12}(p, q_-, k) g_{21}(p, q_-, k_-)$$
$$\times [\frac{1}{\mu_s^2 + 2p_-k} + \frac{1}{\mu_s^2 - 2pk}] a(k) a(k_-). \tag{31}$$

Furthermore, assuming that

$$M_s^{(2)}(\alpha) + \frac{1}{2}[R_{nloc}, V_{nloc}]_{2mes} = 0 \tag{32}$$

with

$$[R_{nloc}, V_{nloc}]_{2mes} = [R_{nloc}, V_{nloc}](a^{\dagger}a)$$
$$+ [R_{nloc}, V_{nloc}](aa) + [R_{nloc}, V_{nloc}](a^{\dagger}a^{\dagger}),$$

we find

$$m_1^{(2)}(k) = \frac{1}{2} \int \frac{d\mathbf{p}}{E_{\mathbf{p}} E_{\mathbf{p}-\mathbf{k}}} [\frac{g_{21}^2(p, q_-, k_-)}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}}$$
$$+ \frac{g_{12}^2(p, q_-, k)}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}}], \tag{33}$$

$$m_2^{(2)}(k) = -\int \frac{d\mathbf{p}}{E_{\mathbf{p}}} g_{12}(p, q_-, k) g_{21}(p, q_-, k_-)$$
$$\times [\frac{1}{\mu_s^2 + 2p_-k} + \frac{1}{\mu_s^2 - 2pk}]. \tag{34}$$

The operators that conserve the boson (antiboson) number can be written as:

$$\frac{1}{2}[R_{nloc}, V_{nloc}](b^{\dagger}b)$$
$$= \int \frac{d\mathbf{k}}{\omega_{\mathbf{k}}} \int \frac{d\mathbf{p}}{E_{\mathbf{p}}^2 E_{\mathbf{p}-\mathbf{k}}} [\frac{g_{11}^2(p, q, k)}{E_{\mathbf{p}} - E_{\mathbf{p}-\mathbf{k}} - \omega_{\mathbf{k}}}$$
$$- \frac{g_{21}^2(p, q_-, k_-)}{E_{\mathbf{p}} + E_{\mathbf{p}-\mathbf{k}} + \omega_{\mathbf{k}}}] b^{\dagger}(p) b(p), \tag{35}$$

$$\frac{1}{2}[R_{nloc}, V_{nloc}](d^\dagger d)$$
$$= \int \frac{d\mathbf{k}}{\omega_\mathbf{k}} \int \frac{d\mathbf{p}}{E^2_\mathbf{p} E_{\mathbf{p}-\mathbf{k}}} [\frac{g^2_{22}(p,q,k)}{E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}}$$
$$- \frac{g^2_{21}(p,q_-,k_-)}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}}] d^\dagger(p) d(p). \quad (36)$$

One can show that from the condition

$$M^{(2)}_b(\alpha) + \frac{1}{2}[R_{nloc}, V_{nloc}]_{2bos} = 0, \quad (37)$$

where

$$[R_{nloc}, V_{nloc}]_{2bos}$$
$$= [R_{nloc}, V_{nloc}](b^\dagger b) + [R_{nloc}, V_{nloc}](b^\dagger d^\dagger)$$
$$+ [R_{nloc}, V_{nloc}](db) + [R_{nloc}, V_{nloc}](d^\dagger d),$$

it follows

$$m^{(2)}_{11}(p) = -\int \frac{d\mathbf{k}}{\omega_\mathbf{k} E_{\mathbf{p}-\mathbf{k}}} [\frac{g^2_{11}(p,q,k)}{E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}}$$
$$- \frac{g^2_{21}(p,q_-,k_-)}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}}], \quad (38)$$

$$m^{(2)}_{22}(p) = -\int \frac{d\mathbf{k}}{\omega_\mathbf{k} E_{\mathbf{p}-\mathbf{k}}} [\frac{g^2_{11}(p,q,k)}{E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}}$$
$$- \frac{g^2_{21}(p,q_-,k_-)}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}}]. \quad (39)$$

Similarly one can obtain the non-diagonal coefficients

$$m^{(2)}_{12}(p) = m^{(2)}_{21}(p)$$
$$= -\int \frac{d\mathbf{k}}{\omega_\mathbf{k} E_{\mathbf{p}-\mathbf{k}}} g_{11}(p,q,k) g_{21}(p,q_-,k_-)$$
$$\times [\frac{1}{E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}} - \frac{1}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}}] \quad (40)$$

or

$$m^{(2)}_{12}(p) = m^{(2)}_{21}(p)$$
$$= -\int \frac{d\mathbf{k}}{\omega_\mathbf{k}} g_{11}(p,q,k) g_{21}(p,q_-,k_-)$$
$$\times [\frac{1}{\mu^2_s - 2pk} + \frac{1}{\mu^2_s + 2p_- k}]$$
$$- \int \frac{d\mathbf{q}}{E_\mathbf{q}} g_{11}(p,q,u) g_{21}(p,q_-,u_-)$$
$$\times (\frac{1}{2[\mu^2_b - pq] - \mu^2_s} + \frac{1}{2[\mu^2_b + pq_-] - \mu^2_s}), \quad (41)$$

where $u = (E_{\mathbf{p}-\mathbf{q}}, \mathbf{p} - \mathbf{q})$.

Thus the clothing procedure has allowed us to get analytical expressions for the interaction operators between the clothed particles. Moreover, we have obtained some prescriptions when finding the coefficients in the "mass renormalization" operators.

At last, one should emphasize that if one starts from expansion

$$H_{nsc}(\mathbf{x}) = \sum_{p=2}^{\infty} H^{(p)}_{nsc}(\mathbf{x}) \quad (42)$$

with the second-order contribution $H^{(2)}_{nsc} = M^{(2)}_s + M^{(2)}_s = 0$, then the RI would be violated at the beginning because of the obvious discrepancy between

$$[H_F, \mathbf{D}^{(2)}] = [\mathbf{N}_F, H^{(2)}_{nsc}] + [\mathbf{N}_B, H_{sc}], \quad (43)$$

and

$$[P_k, D^{(p)}_j] = i\delta_{kj} H^{(p)}_{nsc}, \quad (p = 2, 3, ...). \quad (44)$$

By using previous equations, we obtain

$$-\int \mathbf{x}[H_F, H_{sc}(\mathbf{x})] d\mathbf{x}$$
$$= [H_F, \mathbf{N}_I] + [H_I, \mathbf{N}_I] + [H_{nsc}, \mathbf{N}_F]. \quad (45)$$

Evidently, this equation is fulfilled if we put

$$\mathbf{N}_I = \mathbf{N}_B \equiv -\int \mathbf{x} H_{sc}(\mathbf{x}) d\mathbf{x}, \quad (46)$$

$$[H_{sc}, \mathbf{N}_I] = -\int \mathbf{x} d\mathbf{x} \int d\mathbf{x}' [H_{sc}(\mathbf{x}'), H_{sc}(\mathbf{x})]$$
$$= [\mathbf{N}_F + \mathbf{N}_I, H_{nsc}]. \quad (47)$$

In a model with $H_{nsc} = 0$ the latter reduces to

$$\int e^{-i\mathbf{PX}} \mathbf{I} e^{i\mathbf{PX}} d\mathbf{X} = 0, \quad (48)$$

where

$$\mathbf{I} = \frac{1}{2} \int \mathbf{r} d\mathbf{r} [H_{sc}(\frac{1}{2}\mathbf{r}), H_{sc}(-\frac{1}{2}\mathbf{r})]. \quad (49)$$

One should note that we have arrived to previous equation being inside the Poincaré algebra itself without addressing the Noether integrals.

At this point, we put $\mathbf{N}_I = \mathbf{N}_B + \mathbf{D}$,

$$[H_F, \mathbf{D}] = [\mathbf{N}_B + \mathbf{D}, H_{sc}] + [\mathbf{N}_F + \mathbf{N}_B + \mathbf{D}, H_{nsc}], \quad (50)$$

that replaces commutator $[H, \mathbf{N}] = i\mathbf{P}$ and determines displacement $\mathbf{D}$. Assuming that scalar density $H_{sc}(\mathbf{x})$ is of the first order in coupling constants involved and putting

$$H_{nsc}(\mathbf{x}) = \sum_{p=2}^{\infty} H^{(p)}_{nsc}(\mathbf{x}), \quad (51)$$

we will search operator $\mathbf{D}$ in the form:

$$\mathbf{D} = \sum_{p=2}^{\infty} \mathbf{D}^{(p)}, \quad (52)$$

i.e., as a perturbation expansion in powers of the interaction H_{sc}. Here label (p) denotes the p-th order in these constants. One should keep in mind that higher ($p \geq 2$) terms are usually associated with perturbation series for mass and vertex counterterms.

By substituting H_{nsc} and $\mathbf{D}$ we get the chain of relations:

$$[H_F, \mathbf{D}^{(2)}] = [\mathbf{N}_B, H_{sc}] + [\mathbf{N}_F, H^{(2)}_{nsc}], \quad (53)$$

$$[H_F, \mathbf{D}^{(3)}] = [\mathbf{D}^{(2)}, H_{sc}] + [\mathbf{N}_F, H^{(3)}_{nsc}] + [\mathbf{N}_B, H^{(2)}_{nsc}], \quad (54)$$

$$[P^k, D^{(p)j}] = 0, \qquad (p = 2, 3, \ldots) \tag{55}$$

$$\ldots \qquad \ldots \qquad \ldots \qquad \ldots$$

Further, after such substitutions into the commutators

$$[P_k, N_j] = i\delta_{kj} H,\ [J_k, N_j] = i\varepsilon_{kjl} N_l,$$

$$[N_k, N_j] = -i\varepsilon_{kjl} J_l$$

we deduce, respectively, the following relations:

$$[P_k, D_j^{(p)}] = i\delta_{kj} H_{nsc}^{(p)}, \quad (p = 2, 3, ...) \tag{56}$$

$$[J_k, D_j^{(p)}] = i\varepsilon_{kjl} D_l^{(p)}, \tag{57}$$

$$[N_{Fk}, N_{Bj}] + [N_{Bk}, N_{Fj}] = 0, \tag{58}$$

$$[N_{Fk}, D_j^{(2)}] + [D_k^{(2)}, N_{Fj}] + [N_{Bk}, N_{Bj}] = 0, \tag{59}$$

$$\begin{aligned} &[N_{Fk}, D_j^{(3)}] + [D_k^{(3)}, N_{Fj}] \\ &\quad + [N_{Bk}, D_j^{(2)}] + [D_k^{(2)}, N_{Bj}] = 0, \end{aligned} \tag{60}$$

$$(p = 2, 3, \ldots).$$

6. DISCUSSION. TOWARDS WORKING FORMULAE

We see that our algebraic approach in combination with the UCT method makes our consideration more and more appropriate for practical applications (in particular, as one has to work with the vertex cutoffs). The formulae for the $2 \to 2$ interactions become more tractable if we assume that

$$g_{\varepsilon'\varepsilon}(p', p, k)$$

$$= v_{\varepsilon'\varepsilon}([k + (-1)^{\varepsilon'} p' - (-1)^{\varepsilon} p][k - (-1)^{\varepsilon'} p' + (-1)^{\varepsilon} p]). \tag{61}$$

One can verify the nonlocal model with such cutoffs possesses necessary properties. In terms of the $v_{\varepsilon'\varepsilon}$ functions we get

$$\begin{aligned} m_1^{(2)}(k) = \frac{1}{2} \int \frac{d\mathbf{p}}{E_\mathbf{p} E_{\mathbf{p}-\mathbf{k}}} [&\frac{v_{21}^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}} \\ &+ \frac{v_{12}^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}}], \end{aligned} \tag{62}$$

$$\begin{aligned} m_2^{(2)}(k) = -\int \frac{d\mathbf{p}}{E_\mathbf{p}} &v_{21}(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2) \\ &\times v_{12}(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2) \\ &\times [\frac{1}{\mu_s^2 + 2p\_k} + \frac{1}{\mu_s^2 - 2pk}]. \end{aligned} \tag{63}$$

Now, by handling the charge-independent cutoffs,

$$v_{12}(x) = v_{21}(x) = f(x), \tag{64}$$

we obtain

$$m_1^{(2)}(k) = m_2^{(2)}(k)$$

$$= -\int \frac{d\mathbf{p}}{E_\mathbf{p}} \frac{f^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}+\mathbf{k}})^2)}{\mu_s^2 + 2pk}$$

$$-\int \frac{d\mathbf{p}}{E_\mathbf{p}} \frac{f^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{\mu_s^2 - 2pk}. \tag{65}$$

In other words, the option (64) yields the momentum-independent coefficients $m_1^{(2)}(k) = m_2^{(2)}(k) \equiv m_s^{(2)}$. Indeed, along with the Lorentz invariant denominators the integrand in the r.h.s. of (65) contains function $f(I)$ whose argument

$$I(\mathbf{p}, \mathbf{k}) \equiv \omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2$$

$$= \mu_s^2 - 2\mu_b^2 - 2E_\mathbf{p} E_{\mathbf{p}-\mathbf{k}} - 2\mathbf{p}(\mathbf{p} - \mathbf{k})$$

does not change under the simultaneous transformation $\mathbf{p} \Rightarrow \mathbf{p}' = \mathbf{\Lambda p}$ and $\mathbf{p} - \mathbf{k} \Rightarrow \mathbf{\Lambda}(\mathbf{p} - \mathbf{k})$ on the mass shells $p^2 = \mu_b^2$ and $k^2 = \mu_s^2$. Now, we can reduce the triple integral to the simple one:

$$m_s^{(2)} = 8\pi \int_0^\infty \frac{t^2 dt}{\sqrt{t^2 + \mu_b^2}} \frac{f^2(\mu_s^2 - 4t^2 - 4\mu_b^2)}{4t^2 + 4\mu_b^2 - \mu_s^2}. \tag{66}$$

Furthermore, it has turned out:

$$m_{11}^{(2)}(p) = m_{22}^{(2)}(p)$$

$$\begin{aligned} = -\int \frac{d\mathbf{k}}{\omega_\mathbf{k} E_{\mathbf{p}-\mathbf{k}}} [&\frac{v_{11}^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}})^2)}{E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}} \\ &- \frac{v_{21}^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}}], \end{aligned} \tag{67}$$

$$m_{12}^{(2)}(p) = m_{21}^{(2)}(p)$$

$$\begin{aligned} = -\int \frac{d\mathbf{k}}{\omega_\mathbf{k} E_{\mathbf{p}-\mathbf{k}}} &v_{11}(\omega_\mathbf{k}^2 - (E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}})^2) \\ &\times v_{21}(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2) \\ \times [&\frac{1}{E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}} - \omega_\mathbf{k}} - \frac{1}{E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}} + \omega_\mathbf{k}}]. \end{aligned} \tag{68}$$

Evaluation of these coefficients is simplified once we put

$$v_{11}(\omega_\mathbf{k}^2 - (E_\mathbf{p} - E_{\mathbf{p}-\mathbf{k}})^2) = v_{21}(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)$$

$$= f(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2), \tag{69}$$

$$m_b^{(2)}(p) \equiv m_{11}^{(2)}(p) = m_{21}^{(2)}(p)$$

$$\begin{aligned} &= 2\int \frac{d\mathbf{k}}{\omega_\mathbf{k}} \frac{f^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{E_{\mathbf{p}-\mathbf{k}}^2 - (E_\mathbf{p} - \omega_\mathbf{k})^2} \\ &+ 2\int \frac{d\mathbf{k}}{E_{\mathbf{p}-\mathbf{k}}} \frac{f^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2} \end{aligned} \tag{70}$$

or

$$m_b^{(2)}(p) = C_1(p) + C_2(p),$$

$$C_1(p) = 2\int \frac{d\mathbf{k}}{\omega_\mathbf{k}} \frac{f^2(\omega_\mathbf{k}^2 - (E_\mathbf{p} + E_{\mathbf{p}-\mathbf{k}})^2)}{2pk - \mu_s^2},$$

$$C_2(p) = 2\int \frac{d\mathbf{q}}{E_\mathbf{q}} \frac{f^2(\mu_s^2 - 2\mu_b^2 - 2pq)}{\mu_s^2 - 2\mu_b^2 - 2pq}.$$

Evidently, the second integral does not depend upon p so

$$C_2(p) = C_2(0) = 2\int \frac{d\mathbf{q}}{E_\mathbf{q}} \frac{f^2(\mu_s^2 - 2\mu_b^2 - 2\mu_b E_\mathbf{q})}{\mu_s^2 - 2\mu_b^2 - 2\mu_b E_\mathbf{q}}$$

$$= 8\pi \int_0^\infty \frac{q^2 dq}{E_{\mathbf{q}}} \frac{f^2(\mu_s^2 - 2\mu_b^2 - 2\mu_b E_{\mathbf{q}})}{\mu_s^2 - 2\mu_b^2 - 2\mu_b E_{\mathbf{q}}}. \qquad (71)$$

It is not the case for integral $C_1(p)$. Thus the boson "mass renormalization" coefficients may be momentum dependent.

7. CONCLUSIONS

In order to avoid ultraviolet divergences typical of many field theories we have introduced some covariant cutoff functions in momentum space in the Wentzel field model, that makes our model nonlocal. For this model we retain the property of the interaction density to be Lorentz-scalar.

We have shown how in the framework of the nonlocal meson-boson field model one can build interactions between the clothed mesons and bosons. Moreover, the mass renormalization terms, that are compulsory to ensure the relativistic invariance of the theory as a whole (in Dirac's sense), turn out to be expressed through certain covariant integrals. They are convergent in the field model with appropriate cutoff factors.

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НЕЛОКАЛЬНОЕ РАСШИРЕНИЕ МОДЕЛИ ВЕНТЦЕЛЯ В ПРЕДСТАВЛЕНИИ ОДЕТЫХ ЧАСТИЦ

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Для того чтобы избежать ультрафиолетовых расходимостей, типичных для многих полевых теорий, вводятся ковариантные обрезающие функции в импульсном пространстве в модели Вентцеля. Показано, каким образом в рамках нелокальной мезон-бозонной полевой модели можно построить взаимодействия между одетыми мезонами и бозонами. Кроме того, массовые перенормировочные члены, которые обязательны для обеспечения релятивистской инвариантности теории в целом (по Дираку), оказываются выраженными через определенные ковариантные интегралы. Эти интегралы сходятся в полевой модели с соответствующими обрезающими функциями.

НЕЛОКАЛЬНЕ РОЗШИРЕННЯ МОДЕЛІ ВЕНТЦЕЛЯ У ЗОБРАЖЕННІ ОДЯГНЕНИХ ЧАСТИНОК

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Щоб уникнути ультрафіолетових розбіжностей, типових для багатьох польових теорій, вводяться коваріантні обрезаючі функції в імпульсному просторі в моделі Вентцеля. Показано, як в рамках нелокальної мезон-бозонної польової моделі можна побудувати взаємодії між одягненими мезонами і бозонами. Крім того, масові перенорміровочні члени, які обов'язкові для забезпечення релятивістської інваріантності теорії у цілому (за Діраком), виявляються вираженими через певні коваріантні інтеграли. Дані інтеграли збігаються в польовій моделі з відповідними обрезаючими функціями.