RENORMALIZATION THE QUANTUM FIELD MODEL OF PARTICLE INTERACTION

V.I. Tikhonov [1*] ***and A.V. Tykhonov*** [2]

[1] *Odessa National Academy of Telecommunication n.a. A.S.Popov, 65029, Odessa, Ukraine*

[2] *Odessa National Polytechnic University, 65044, Odessa, Ukraine*

(Received October 31, 2011)

The model simulates the interaction of abstract entities distinguished in a physical experiment and denoted as particles. Empirical data results in the non-hermitian anti-symmetric matrix of particle relationship. The real and imaginary parts of the matrix correspond to symmetric and asymmetric coupling of particles. The relationship matrix evolves to multiplication of pure defined hermitian metric tensor and curvature vector. The real spectrum of metric tensor extended into the complex space with invariant spectrum power results in renormalized non-singular quantum field model of particle interaction.

PACS: 11.10.Gh, 11.10.Nx

1. INTRODUCTION

The classic field theory (CFT) studies the real and complex functions (scalar fields, vector fields, tensor fields) presented in a priory pre-defined linear spaces [1]. This approach is truly relevant to the certain universe stratum available for the human perception. The enhanced experiments extend the scope of analysis into macro- and microspheres, though the cognitive horizon remains infinitesimal part of the Universe. The macro-world violates naive insights of plane Euclidian space. The micro-world drastically changes the understanding of what is the space eventually; it appears multidimensional, non-orthonormal and asymmetric.

In contrast to CFT, the quantum field theory (QFT) studies the models of abstract spaces simulating the physical experiments [2, 3]. In this work, authors try a holistic approach to the modeling of arbitrary physical entities emerged in stochastic experiment (called particles). The model assimilates the space curvature and asymmetry properties of the empirical data.

2. THE RELATIONSHIP MATRIX OF EMPIRICAL DATA

Let X be an arbitrary open set of N interacting elements $(x_1, ..., x_N) \in X$, distinguished in the physical experiment within a certain time interval. These elements we denote as particles. Any $x_n \in X$ consider open, e.g. it may interact with particles of X and out of X (latent interaction). The interaction of particles may have the asymmetry. Let Q be the non-hermitian complex anti-symmetric matrix of particle's relationship. We define a binary operation $\otimes$ to pick hermitian matrix H out of the non-hermitian matrix Q:

$$\begin{aligned} Q &= H \otimes \psi, \\ \psi &= \{\psi_n\}, \end{aligned} \tag{1}$$

where $\psi_n = e^{i\theta(n)}$ is the vector of curvature for matrix Q. The operator $\otimes$ multiplies any diagonal entry $Q(n,n)$ by the $\psi(n)$. The main diagonal of Q with complex numbers evolves to real numbers and non-hermitian matrix Q results in hermitian matrix H:

$$H = Q \otimes \psi^* = Q \otimes e^{-i\theta(n)}, \tag{2}$$

where "*" is the complex conjugation symbol. We will utilize the irreducible representation of hermitian matrix H in eigenvector basis Z [4]:

$$\begin{aligned} H &= Z^* \cdot \Lambda \cdot Z, \\ \Lambda &= I \otimes \lambda, \end{aligned} \tag{3}$$

where Λ is the diagonal matrix of eigenvalues λ_n, I is the unit diagonal matrix, and $\lambda = \{\lambda_n\}$ is the vector of eigenvalues λ_n (also called spectrum of H). If some eigenvalues λ_n are not positive, the matrix H is poorly defined; it plagues singularity and needs renormalization [5, 6].

3. RENORMALIZATION OF METRIC TENSOR

To renormalize matrix H we make three steps.

Step 1. Suppose eigenvectors z_n and eigenvalues λ_n are ordered by λ_n decrement (if not they are to be re-indexed):

$$\lambda_1 \geq \lambda_2 \geq ...\lambda_n \geq ...\lambda_N. \tag{4}$$

*Corresponding author E-mail address: victor.tykhonov@onat.edu.ua

Step 2. Let $\alpha = |\lambda_n|_{\max}$ be the maximal module of λ_n, $n = 1, 2, ..., N$. Evolve spectrum $\lambda_n = \{\lambda_n\}$ to the spectrum $\beta = \{\beta_n\}$ where $|\beta_n|_{\max} = 1$:

$$\beta_n = \lambda_n / \alpha. \tag{5}$$

Now, the spectrum β and matrix Q become dimensionless.

Step 3. Consider spectrum $\beta = \{\beta_n\}$ being the real part of the complex spectrum $\rho = \{\rho_n\}$, where $\rho_n = \exp(i \cdot \varphi_n)$:

$$\beta_n = \mathrm{Re}(\rho_n) = \mathrm{Re}[\exp(i \cdot \varphi_n)] = \cos(\varphi_n). \tag{6}$$

Let $\Omega_R = I \otimes \beta$ and $C_R = f(H) = Z^* \cdot \Omega_R \cdot Z$ being the real part of an abstract complex $C = f(H)$; f symbolizes the functional dependence on the argument. The imaginary part of the complex spectrum is a vector $\gamma = \{\gamma_n\}$, where

$$\gamma_n = \mathrm{Im}(\rho_n) = \mathrm{Im}[\exp(i \cdot \varphi_n)] = \sin(\varphi_n). \tag{7}$$

Let $\Omega_I = I \otimes \gamma$. The imaginary part of C is $C_I = Z^* \cdot \Omega_I \cdot Z$. Thus, we have

$$\begin{aligned} C &= C_R + i \cdot C_I = Z^* \cdot \Omega_R \cdot Z + Z^* \cdot \Omega_I \cdot Z \\ &= (Z^* \cdot \Omega_R + Z^* \cdot \Omega_I) \cdot Z = Z^* \cdot (\Omega_R + \Omega_I) \cdot Z \\ &= Z^* \cdot \Omega \cdot Z; \end{aligned} \tag{8}$$

$$\begin{aligned} &C = Z^* \cdot \Omega \cdot Z, \\ &\Omega = \Omega_R + i \cdot \Omega_I = I \otimes (\beta + i \cdot \gamma) \\ &\quad = I \otimes \exp(i \cdot \varphi) = I \otimes \phi, \\ &\varphi = \{\varphi_n\}, \quad \phi = \exp(\varphi). \end{aligned} \tag{9}$$

The matrix $C = f(H)$ we denote as a *renormalized complex metric tensor (RMT) of the particle interaction* for the empirical relationship matrix $Q = H \otimes \psi$. Matrix C is non-singular as the inverse matrix C^{-1} always exists: $C^{-1} = C^*$. We have:

$$\begin{aligned} C \cdot C^{-1} &= (Z^* \cdot \Omega \cdot Z) \cdot (Z^* \cdot \Omega \cdot Z)^* \\ &= (Z^* \cdot \Omega \cdot Z) \cdot (Z^* \cdot \Omega^* \cdot Z) \\ &= (Z^* \cdot \Omega \cdot (Z \cdot Z^*) \cdot \Omega^* \cdot Z) \\ &= Z^* \cdot (\Omega \cdot I \cdot \Omega^*) \cdot Z = Z^* \cdot I \cdot Z = I. \end{aligned} \tag{10}$$

4. RENORMALIZATION THE QUANTUM FIELD MODEL

We will define the multiplication

$$G = f(Q) = C \otimes \psi = (Z \cdot I \otimes \phi \cdot Z^*) \otimes \psi. \tag{11}$$

as renormalized complex presentation of the empirical data in form of relationship matrix Q. Three objects present $G = f(Q)$:

$$G = f(Q) = \{Z,\ \phi,\ \psi\}, \tag{12}$$

Z is the 2-valence one-covariant and one contra-variant unitary matrix operator, ϕ is the 1-valence co-variant complex phase vector, and ψ is the 1-valence covariant complex curvature vector.

Therefore, the multiplication (9) is 4-valence 3-co-variant and 1-contra-variant tensor of metrics and curvature. We denote the system $G = \{Z,\ \phi,\ \psi\}$, in respect to the multiplication (9), as co-variant *renormalized quantum field model (RQM) of particle interaction* for the empirical relationship matrix $Q = H \otimes \psi$. The system $\overline{G} = \{Z^*,\ \phi^*,\ \psi^*\}$ we denote as contra-variant RQM-model towards G. Obviously, $\overline{G}$ is 4-valence 1-co-variant and 3-contra-variant tensor.

Consider the obvious properties of the operator $\otimes$ spoken above:

$$\begin{aligned} &\psi \otimes \psi^* = \psi^* \otimes \psi = e = \{e_1, e_2, ..., e_n, ..., e_N\} \\ &\quad = \{1, 1, ..., 1\}; \\ &C \otimes \psi \equiv \psi \otimes C; \\ &C \otimes e \equiv C; \\ &\psi \otimes I \otimes \psi^* = \psi^* \otimes I \otimes \psi = I. \end{aligned} \tag{13}$$

The invariant tensor of RQM-model we define as tripled convolution

$$\begin{aligned} G \cdot \overline{G} &= (C \otimes \psi) \cdot (C^* \otimes \psi^*) \\ &= \psi \otimes (C \cdot C^*) \otimes \psi^* = \psi \otimes I \otimes \psi^* = I. \end{aligned} \tag{14}$$

The matrix I is 2-valence one-co-variant and one-contra-variant tensor (neutral operator). It is clear that tensor multiplication (13) is commutative:

$$G \times \overline{G} \equiv \overline{G} \times G = I. \tag{15}$$

5. THE QUANTUM FIELD MODEL ANALYSIS

We will discuss some properties and special cases of the renormalized quantum field model (RQM) of particle interaction.

Property 1. The RQM has $\eta = N \cdot (N+1) - 1$ degrees of freedom.

In fact, any square complex matrix Q with complex main diagonal and anti-symmetric entries $Q(n,m) = \overline{Q(m,n)}$ has $N \times N + N = N \cdot (N+1)$ independent entries, e.g. freedom degrees. The renormalization procedure given above provides: 1 freedom degree limitation ($|\beta_n|_{\max} = |\lambda_n|_{\max}/\alpha = 1$); N freedom degrees extension (Ω_I) and N freedom degrees limitation ($\Omega_I \cdot \Omega_R = I \otimes \exp(i \cdot \varphi) = I \otimes \phi$). Eventually $\eta = N \cdot (N+1) - 1$. Consider normalization factor α, the *particular renormalized quantum field model (PRM)* $G_\alpha = \alpha \cdot G$ results in $\eta = N \cdot (N+1)$ freedom degrees.

Property 2. In the 4-dimensional space ($N = 4$) the PRM-model has 20 freedom degrees.

The same number $\mu = N^2 \cdot (N^2 - 1)/12 = 20$ of freedom degrees results in the 4-valence 3-covariant and 1-contra-variant Riemann curvature tensor [2]. That means that the 4-dimensional *PRM* model is isomorphic to the 4-dimensional space presented by the Riemann curvature tensor. These two presentations bijectively map each other.

Case 1: $\psi = \mathbf{e}$ (no curvature). The RQM evolves into the normalized spectral phase C-filter in Z-basis:

$$G \rightarrow C = Z^* \cdot \Omega \cdot Z = Z \cdot (I \otimes \phi) \cdot Z^*. \quad (16)$$

The filtering procedure for any sample $y = \{y_1, y_2, ..., y_n\}$ is $y_C = C \cdot y$. Manipulating the factor $\alpha = |\lambda_n|_{\max}$ we obtain the general spectral phase filter

$$C_\alpha = \alpha \cdot (F^* \cdot \Omega \cdot F). \quad (17)$$

The $\alpha > 1$ amplifies the y-output; $\alpha < 1$ suppresses the y-output.

Property 3. The C-filter (16) is spinor [3].
In special case $\phi = \{\exp(i \cdot \pi \cdot m)_n\}$ all the diagonal entries of matrix $\Omega = I \otimes \phi$ turn into the alternating units ± 1. We denote the correspondent vector $\mathbf{s}$ as *signature* and matrix S as *signature filter* (S-filter):

$$\begin{aligned} \mathbf{s} &= \{s_n\} = \{\exp(i \cdot \pi \cdot m)_n\}, \\ S &= I \otimes \mathbf{s}. \end{aligned} \quad (18)$$

Case 2: $\psi = \mathbf{e}$ (no curvature); $\alpha = 1$; $Z \rightarrow M_S = [\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{t}]$ are real Euclidian orts of the (3+1)-dimensional orthonormal space; $\mathbf{s} \rightarrow \mathbf{s_M} = [-1, 1, 1, 1]$. The $\mathbf{s_M}$ is the signature of the Minkovski space M_S [7]:

$$C \rightarrow M_S^* \cdot \mathbf{s_M} \cdot M_S = I \otimes \mathbf{s_M}. \quad (19)$$

Case 3: $\psi = \mathbf{e}$; $\alpha = 1$; $Z \rightarrow F$ is the Fourier basis. Now, the C-filter (16) turns into the conventional digital phase Fourier filter Φ [8]:

$$C \rightarrow \Phi = F^* \cdot \Omega \cdot F. \quad (20)$$

The Fourier filtering for any sample $y = \{y_1, y_2, ..., y_n\}$ is $y_C = \Phi \cdot y$.

Here $F \cdot y = y_F$ is direct Fourier transform; y_F is Fourier image; the $\Omega \cdot y_F = y_{F\Omega}$ is spectral correction of Fourier image y_F; the $F^* \cdot y_{F\Omega} = y_C$ is reverse Fourier transform for the corrected Fourier image $y_{F\Omega}$.

Case 4: $\psi = \mathbf{e}$; $\alpha = 1$; $Z \rightarrow F$; $\Omega \rightarrow \Omega_R = \mathrm{Re}(\Omega)$. Now, the Φ-filter (20) turns into the passive spectral density Fourier filter: $\Phi \rightarrow \Phi_R = (F \cdot \Omega_R \cdot F^*)$. Manipulating the factor $\alpha = |\lambda_n|_{\max}$ we obtain the general spectral density Fourier filter

$$\Phi_{R\alpha} = \alpha \cdot (F \cdot \Omega_R \cdot F^*). \quad (21)$$

Property 4. Take equation (8), next derive $G = C \otimes \psi = (C_R + i \cdot C_I) \otimes \psi$.
Therefore, we have

$$\begin{aligned} G &= QM + i\, QF, \\ QM &= C_R \otimes \psi, \\ QF &= C_I \otimes \psi, \end{aligned} \quad (22)$$

where QM denotes abstract *"Quantum Matter"*, and QF is abstract *"Quantum Field"*. The composition $G = QM + i\, QF$ has invariant total abstract power P_G obtained from (15):

$$P_G = f(G) = \mathrm{Tr}(G \times \overline{G}) = I = \mathrm{Tr}(I) = N, \quad (23)$$

where Tr is the matrix trace symbol.

6. CONCLUSIONS

The work studies the interaction model for an arbitrary set of abstract physical entities called particles. The simulation model assumes the statistical experiment output data in form of non-hermitian anti-symmetric complex matrix with non-real main diagonal entries. To dissimulate the complexity of the matrix diagonal a special binary operation established in the work. Due to this operation, the non-hermitian data matrix evolves into the composition of the two components: hermitian metrical matrix of linear complex space and the curvature vector. Hence, these two components are treated individually.

To override the inherent singularity of hermitian matrix, the new axiom is applied: the spectrum of a unitary operator metrics is a real part of the complex spectrum defined for extended non-unitary operator metrics. The extended complex spectrum considered has constant power and is invariant to the phase rotations (non-singular). From that point of view, the concept of quantum matter and quantum field component's composition is originated in the work.

References

1. J.J. Binney. *Classical Fields. Part I: Relativistic Covariance.* "Oxford University Trinity Term", 1999, 79 p.

2. R. Penrose. *Structure of space-time. Lectures in Mathematics and Physics Chapter VII.* New York-Amsterdam: "W.A. Benjamin, Inc.", 1968, 520 p.

3. U.B. Rumer, A.I. Fet. *Group Theory and Quantum Fields.* Moscow: "Nauka", 1977, 247 p. (in Russian).

4. F.R. Hantmaher. *The Matrix Theory* / 2nd edition. Moscow: "Nauka", 1966, 576 p. (in Russian).

5. R.P. Feynman. *The Strange Theory Of Light And Matter.* Princeton, New Jersey: "Princeton University Press", 2006, 158 p.

6. N.N. Bogolubov, D.V. Shyrkov. *Introduction into the Quantum Field Theory.* Moscow: "Nauka", 1984, 600 p. (in Russian).

7. H. Minkowski. Basic Equations for Electromagnetic Processes in Moving Objects // http://jvr.freewebpage.org/TableOfContents/Volume4/Issue4/MinkowskiHermann.pdf (in German).

8. S.W. Smith. *The Scientist and Engineer's Guide to Digital Signal Processing. Second Edition.* San Diego: "California Technical Publishing", 1999, 643 p.

РЕНОРМАЛИЗАЦИЯ КВАНТОВО-ПОЛЕВОЙ МОДЕЛИ ВЗАИМОДЕЙСТВИЯ ЧАСТИЦ

В.И. Тихонов, А.В. Тихонов

Рассматривается модель взаимодействия абстрактных сущностей, различимых в физическом эксперименте и названных частицами. Эмпирические данные представлены в форме неэрмитовой антисимметрической матрицы взаимодействия частиц. Вещественные и мнимые элементы матрицы соответствуют симметрической и асимметрической составляющим взаимодействующей пары частиц. Матрица взаимодействия приведена к произведению слабо обусловленного эрмитового метрического тензора на вектор кривизны. Вещественный спектр метрического тензора расширен в комплексную область с условием инвариантности мощности спектра, в результате чего получена ренормализованная несингулярная квантово-полевая модель взаимодействия частиц.

РЕНОРМАЛІЗАЦІЯ КВАНТОВО-ПОЛЬОВОЇ МОДЕЛІ ВЗАЄМОДІЮЧИХ ЧАСТОК

В.І. Тіхонов, А.В. Тихонов

Розглянуто модель взаємодії абстрактних сутностей, помітних у фізичному експерименті і названих частками. Емпіричні дані представлені у вигляді неермітової антисиметричної матриці взаємодії часток. Дійсні та уявні елементи матриці відповідають симетричній та асиметричній складовим взаємодіючої пари часток. Матриця взаємодії приведена к добутку слабо обумовленого метричного тензора на вектор кривизни. Дійсний спектр метричного тензора розширено у комплексну область за умови інваріантності потужності спектру, в результаті чого отримано ренормалізовану несингулярну квантово-польову модель взаємодії часток.