$AdS_4 \times CP^3$ SUPERSTRING AS $OSp(4|6)/(SO(1,3) \times U(3))$ SIGMA-MODEL IN CONFORMAL BASIS

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(Received October 26, 2011)

Basic features of Lagrangian formulation for $AdS_4 \times CP^3$ superstring in the framework of $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model approach are reviewed with the emphasis on realization of $osp(4|6)$ background isometry superalgebra as $D = 3$ $\mathcal{N} = 6$ superconformal algebra.

PACS: 11.25.-w; 11.30.Pb; 11.25.Tq

1. INTRODUCTION

Application of relativistic string models to description of non-Abelian gauge theories has rather long history going back to the dual resonance models [1] and 't Hooft approach [2]. The first explicit proposal of duality between gauge theory and string theory was put forward only in 1997 by Maldacena [3] and intensely exploits the concept of supersymmetry [4, 5]. Since then many other examples of gauge fields/strings dualities were proposed (see, e.g. review [6] and references therein), however none of them has been proved from the first principles yet. Even their verification appears a non-trivial task because of non-linearity of the theories involved. To date pivotal role in testing gauge fields/strings dualities is played by (super)symmetry considerations. For that reason to the best explored cases belong highly (super)symmetric ones [3, 7].

This note is devoted to reviewing some aspects of classical string theory related to the Aharony-Bergman-Jafferis-Maldacena (ABJM) duality [7] conjecturing that $D = 3$ $\mathcal{N} = 6$ Chern-Simons-matter theory admits in the 't Hooft limit [2] dual description as the Type IIA superstring on $AdS_4 \times CP^3$ superbackground. This background is known [8] to solve equations of IIA supergravity and preserves 24 of 32 space-time supersymmetries, i.e. corresponding Killing spinor equation has 24 independent solutions out of 32 *maximally* allowed. Background symmetry group is isomorphic to $OSp(4|6)$ supergroup that manifests itself as $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the action of dual Chern-Simons-matter theory. This distinguishes ABJM duality from the AdS_5/CFT_4 one [3] relating $D = 4$ $\mathcal{N} = 4$ super-Yang-Mills theory and the Type IIB string theory on $AdS_5 \times S^5$ superbackground that is *maximally* supersymmetric solution of IIB supergravity. $PSU(2,2|4)$ symmetry of the $AdS_5 \times S^5$ superbackground matches $D = 4$ $\mathcal{N} = 4$ superconformal symmetry of the action of super-Yang-Mills theory.

Symmetry arguments also governed construction of the classical action for $AdS_5 \times S^5$ superstring. It was observed [3] that both parts of the background represent symmetric spaces $AdS_5 = SO(2,4)/SO(1,4)$ and $S^5 = SO(6)/SO(5)$ with isometry groups constituting bosonic subgroup $SO(2,4) \times SU(4)$ of the $PSU(2,2|4)$ supergroup and the number of fermionic generators of $PSU(2,2|4)$ equals 32 that is the Grassmann-odd dimension of superspace. This hinted to identify $AdS_5 \times S^5$ superspace as the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset manifold and the $AdS_5 \times S^5$ superstring action was constructed as the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ supercoset sigma-model [9, 10]. It was then found that the superstring Lagrangian can be presented as quadratic polynomial in Cartan forms associated with the $psu(2,2|4)/(so(1,4) \times so(5))$ supercoset generators using their decomposition into eigenspaces of the discrete Z_4 automorphism of $psu(2,2|4)$ superalgebra [11, 12]. Resulting action is invariant under global $PSU(2,2|4)$ supersymmetry, as well as gauge $SO(1,4) \times SO(5)$ and κ−symmetries and describes correct number of physical degrees of freedom. Moreover, corresponding equations of motion are classically integrable and can be obtained from the zero-curvature condition for the associated Lax connection [13].

Above consideration can be generalized to the $AdS_4 \times CP^3$ superstring case. Namely, $AdS_4 = SO(2,3)/SO(1,3)$ and $CP^3 = SU(4)/U(3)$ are symmetric spaces and their isometry groups can be combined into bosonic subgroup $SO(2,3) \times SO(6)$ of the $OSp(4|6)$ supergroup that also includes 24 fermionic generators equal in number to the supersymmetries preserved by $AdS_4 \times CP^3$ back-

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ground. The $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space has 10 space-time and 24 fermionic directions lacking 8 fermionic directions to span full $AdS_4 \times CP^3$ superspace. Nonetheless it is possible to construct the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type action following the straight-forward generalization of the prescription used to obtain $PSU(2,2|4)/(SO(1,4) \times SO(5))$ sigma-model [9–12]. In [14,15] there was presented the general structure of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model including corresponding equations of motion, $\kappa-$symmetry transformation rules and the Lax connection. By choosing the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset representative explicit form of the Lagrangian was found to the second order in the world-sheet fields and checked against known Penrose limit Lagrangian [16].

In Ref. [17] we have presented the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type action in conformal basis for Cartan forms relying on the $osp(4|6)$ superalgebra realization as the $D = 3$ $\mathcal{N} = 6$ superconformal algebra. The proof was also given in the $SO(1,2) \times SU(3)$ covariant way that among 24 fermionic equations of motion there are only 16 independent for non-singular superstring configurations implying 8-parameter $\kappa-$symmetry of the action. Besides that we have obtained explicit form of the sigma-model-type Lagrangian to all orders in the space-time and anticommuting coordinates for the $OSp(4|6)/(SO(1,3) \times U(3))$ representative compatible with conformal structure. Such a choice allows to formulate stringy side of the duality in terms of coordinates that contain those parametrizing $D = 3$ $\mathcal{N} = 6$ boundary superspace, where the ABJM gauge theory [7] can be formulated aiming at getting new insights into the relation between both theories.

The $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space is the subspace of the full-fledged $AdS_4 \times CP^3$ superspace. Hence the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type action can be obtained by partial $\kappa-$symmetry gauge fixing of the complete $AdS_4 \times CP^3$ superstring action [18]. It is amenable to describe all possible classical string configurations, in particular those that cannot be considered within the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model [14, 18], however it does not correspond to a supercoset sigma-model and the issue of its integrability remains open [19].

2. $OSp(4|6)/(SO(1,3) \times U(3))$ SIGMA-MODEL IN CONFORMAL BASIS

The $osp(4|6)$ superalgebra g of $AdS_4 \times CP^3$ superbackground is isomorphic to $D = 3$ $\mathcal{N} = 6$ superconformal algebra spanned by

$$g = \left\{ \begin{array}{c} M_{mn},\ P_m,\ K_m,\ D \\ V_a{}^b,\ V_a,\ V^a \\ Q^a_\mu,\ \bar{Q}_{\mu a},\ S^{\mu a},\ \bar{S}^\mu_a \end{array} \right\}. \quad (1)$$

Generators in the first line are that of $D = 3$ conformal group, the second line contains generators of the $SU(4) \sim SO(6)$ isometry group of CP^3 manifold. Remaining fermionic generators have been divided into two sets of 12 associated with Poincare and conformal supersymmetries. We adhere to the notations of [17], namely small Latin letters from the middle of the alphabet $k, l, m, n = 1, 2, 3$ label $SO(1,2)$ vectors, while that from the beginning of the alphabet $a, b, c = 1, 2, 3$ label objects transforming in (anti)fundamental representation of $SU(3) \subset SU(4)$. Small Greek letters μ, ν, λ stand for 2-component spinor indices of $Spin\ (1,2)$. (Anti)commutation relations of $D = 3$ $\mathcal{N} = 6$ superconformal algebra can be found in [17].

Crucial role in constructing $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type Lagrangian is played by Z_4 automorphism Ω of the $osp(4|6)$ superalgebra under which all the generators divide into 4 eigenspaces with different eigenvalues

$$g = g_0 \oplus g_1 \oplus g_2 \oplus g_3 : \ \Omega(g_\mathrm{k}) = i^\mathrm{k} g_\mathrm{k}. \quad (2)$$

Invariant eigenspace g_0 is spanned by $so(1,3) \oplus u(3)$ stability algebra generators. g_2 Eigenspace contains $so(2,3)/so(1,3) \oplus su(4)/u(3)$ coset generators and g_1 and g_3 – fermionic ones. In terms of the generators of $D = 3$ $\mathcal{N} = 6$ superconformal algebra these eigenspaces can be realized as

$$\begin{array}{rcl} g_0 & = & \{M_{mn},\ P_m - K_m,\ V_a{}^b\}, \\ g_2 & = & \{D,\ P_m + K_m,\ V_a,\ V^a\}, \\ g_1 & = & \{Q^a_\mu + iS^a_\mu,\ \bar{Q}_{\mu a} - i\bar{S}_{\mu a}\}, \\ g_3 & = & \{Q^a_\mu - iS^a_\mu,\ \bar{Q}_{\mu a} + i\bar{S}_{\mu a}\}. \end{array} \quad (3)$$

Left-invariant Cartan forms $C(d)$ in conformal basis admit the decoposition:

$$\begin{array}{rcl} C(d) & = & G^{-1}dG = \omega^m(d)P_m + c^m(d)K_m + \Delta(d)D \\ & + & G^{mn}(d)M_{mn} + \Omega^a(d)V_a + \Omega_a(d)V^a \\ & + & \Omega_a{}^b(d)V_b{}^a + \omega^\mu_a(d)Q^a_\mu + \bar{\omega}^{\mu a}(d)\bar{Q}_{\mu a} \\ & + & \chi_{\mu a}(d)S^{\mu a} + \bar{\chi}^a_\mu(d)\bar{S}^\mu_a, \end{array} \quad (4)$$

where $G \in OSp(4|6)/(SO(1,3) \times U(3))$ is a supercoset representative. Above division of the $osp(4|6)$ generators under Ω automorphism induces corresponding division of Cartan forms

$$C(d) = C_0(d) + C_2(d) + C_1(d) + C_3(d) \quad (5)$$

with individual summands defined as

$$\begin{array}{rcl} C_0(d) & = & \frac{1}{2}(\omega^m(d) - c^m(d))(P_m - K_m) \\ & + & G^{mn}(d)M_{mn} + \Omega_a{}^b(d)V_b{}^a; \end{array} \quad (6)$$

$$\begin{array}{rcl} C_2(d) & = & \frac{1}{2}(\omega^m(d) + c^m(d))(P_m + K_m) \\ & + & \Delta(d)D + \Omega_a(d)V^a + \Omega^a(d)V_a; \end{array} \quad (7)$$

$$\begin{array}{rcl} C_1(d) & = & \frac{1}{2}(\omega^\mu_a(d) + i\chi^\mu_a(d))(Q^a_\mu + iS^a_\mu) \\ & + & \frac{1}{2}(\bar{\omega}^{\mu a}(d) - i\bar{\chi}^{\mu a}(d))(\bar{Q}_{\mu a} - i\bar{S}_{\mu a}); \end{array} \quad (8)$$

$$\begin{array}{rcl} C_3(d) & = & \frac{1}{2}(\omega^\mu_a(d) - i\chi^\mu_a(d))(Q^a_\mu - iS^a_\mu) \\ & + & \frac{1}{2}(\bar{\omega}^{\mu a}(d) + i\bar{\chi}^{\mu a}(d))(\bar{Q}_{\mu a} + i\bar{S}_{\mu a}). \end{array} \quad (9)$$

Global $OSp(4|6)$ transformations act on the supercoset representative according to the rule

$$\mathsf{L}G = G'\mathsf{H}, \quad \mathsf{L} \in OSp(4|6) \qquad (10)$$

with $\mathsf{H} \in SO(1,3) \times U(3)$ being the compensating coordinate-dependent rotation under which Cartan forms from eigenspaces 1,2 and 3 transform homogeneously

$$C'_{1,2,3} = \mathsf{H} C_{1,2,3} \mathsf{H}^{-1}, \qquad (11)$$

whereas C_0 Cartan forms transform as $SO(1,3) \times U(3)$ connection

$$C'_0 = \mathsf{H} C_0 \mathsf{H}^{-1} - \mathsf{H}^{-1} d\mathsf{H}. \qquad (12)$$

The $OSp(4|6)$ and Z_4-invariant sigma-model-type action [14] is constructed out of Cartan forms $C_{1,2,3}$ according to the general prescription [11–13]. In conformal basis for Cartan forms it is brought to the form [17]

$$\begin{aligned} S &= -\tfrac{1}{2} \int d^2\xi \sqrt{-g} g^{ij} \left[\tfrac{1}{4} (\omega_i^m + c_i^m)(\omega_{jm} + c_{jm}) \right. \\ &+ \left. \Delta_i \Delta_j + \tfrac{1}{2} (\Omega_{ia} \Omega_j^a + \Omega_{ja} \Omega_i^a) \right] + S_{WZ} \end{aligned} \qquad (13)$$

with the Wess-Zumimo action given by

$$\begin{aligned} S_{WZ} &= -\tfrac{1}{4} \varepsilon^{ij} \int d^2\xi \left[(\omega_{ia}^\mu + i\chi_{ia}^\mu) \varepsilon_{\mu\nu} (\bar{\omega}_j^{\nu a} + i\bar{\chi}_j^{\nu a}) \right. \\ &+ \left. (\omega_{ia}^\mu - i\chi_{ia}^\mu) \varepsilon_{\mu\nu} (\bar{\omega}_j^{\nu a} - i\bar{\chi}_j^{\nu a}) \right] \\ &= -\tfrac{1}{2} \varepsilon^{ij} \int d^2\xi \big(\omega_{ia}^\mu \varepsilon_{\mu\nu} \bar{\omega}_j^{\nu a} + \chi_{i\mu a} \varepsilon^{\mu\nu} \bar{\chi}_{j\nu}^a \big). \end{aligned} \qquad (14)$$

The construction implies identification of Cartan forms $\omega^m(d) + c^m(d)$, $\Delta(d)$ and $\Omega_a(d)$, $\Omega^a(d)$ with tangent to AdS_4 and CP^3 components of the supervielbein of $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset manifold. Analogously Cartan forms $\omega_a^\mu(d) + i\chi_a^\mu(d)$, $\omega_a^\mu(d) - i\chi_a^\mu(d)$ and c.c. are identified with the fermionic components of supervielbein.

Equations of motion resulting from variation of the supervielbein bosonic components tangent to AdS_4 and CP^3 parts of the background read

$$\begin{aligned} &\partial_i(\sqrt{-g} g^{ij} (\omega_j^m + c_j^m)) + 2\sqrt{-g} g^{ij} G_i^{mn} (\omega_{jn} \\ &+ c_{jn}) + 2\sqrt{-g} g^{ij} (c_i^m - \omega_i^m) \Delta_j \\ &+ i\varepsilon^{ij} (\omega_{ia}^\mu + i\chi_{ia}^\mu) \sigma_{\mu\nu}^m (\bar{\omega}_j^{\nu a} - i\bar{\chi}_j^{\nu a}) \\ &- i\varepsilon^{ij} (\omega_{ia}^\mu - i\chi_{ia}^\mu) \sigma_{\mu\nu}^m (\bar{\omega}_j^{\nu a} + i\bar{\chi}_j^{\nu a}) = 0, \end{aligned} \qquad (15)$$

$$\begin{aligned} &\partial_i(\sqrt{-g} g^{ij} \Delta_j) + \tfrac{1}{2} \sqrt{-g} g^{ij} (\omega_i^m - c_i^m)(\omega_{jm} \\ &+ c_{jm}) + \tfrac{1}{2} \varepsilon^{ij} (\omega_{ia}^\mu + i\chi_{ia}^\mu) \varepsilon_{\mu\nu} (\bar{\omega}_j^{\nu a} - i\bar{\chi}_j^{\nu a}) \\ &+ \tfrac{1}{2} \varepsilon^{ij} (\omega_{ia}^\mu - i\chi_{ia}^\mu) \varepsilon_{\mu\nu} (\bar{\omega}_j^{\nu a} + i\bar{\chi}_j^{\nu a}) = 0 \end{aligned} \qquad (16)$$

and

$$\begin{aligned} &\partial_i(\sqrt{-g} g^{ij} \Omega_j^a) + i\sqrt{-g} g^{ij} \Omega_i^b (\Omega_{jb}{}^a + \delta_b^a \Omega_{jc}{}^c) \\ &- \tfrac{i}{2} \varepsilon^{ij} \varepsilon^{abc} (\omega_{ib}^\mu + i\chi_{ib}^\mu) \varepsilon_{\mu\nu} (\omega_{jc}^\nu + i\chi_{jc}^\nu) \\ &- \tfrac{i}{2} \varepsilon^{ij} \varepsilon^{abc} (\omega_{ib}^\mu - i\chi_{ib}^\mu) \varepsilon_{\mu\nu} (\omega_{jc}^\nu - i\chi_{jc}^\nu) = 0 \end{aligned} \qquad (17)$$

respectively. Similarly fermionic equations of motions can be cast into the form

$$\begin{aligned} &V_+^{ij} [(\sigma_{m\mu\nu} (\omega_i^m + c_i^m) + 2i\varepsilon_{\mu\nu} \Delta_i)(\omega_{ja}^\nu + i\chi_{ja}^\nu) \\ &+ 2\varepsilon_{abc} \Omega_i^b (\bar{\omega}_j^{\mu c} - i\bar{\chi}_j^{\mu c})] = 0 \end{aligned} \qquad (18)$$

for Cartan forms from the C_1 eigenspace (8) and

$$\begin{aligned} &V_-^{ij} [(\sigma_{m\mu\nu} (\omega_i^m + c_i^m) - 2i\varepsilon_{\mu\nu} \Delta_i)(\omega_{ja}^\nu - i\chi_{ja}^\nu) \\ &- 2\varepsilon_{abc} \Omega_i^b (\bar{\omega}_j^{\mu c} + i\bar{\chi}_j^{\mu c})] = 0 \end{aligned} \qquad (19)$$

for those from the C_3 eigenspace (9) and c.c. equations. $V_\pm^{ij} = \frac{1}{2}(\sqrt{-g} g^{ij} \pm \varepsilon^{ij})$ are projectors allowing to split world-sheet vectors and tensors into (anti)self-dual parts. Common feature of the Green-Schwarz superstring models is that not all fermionic equations are independent. This implies via the second Noether theorem invariance of the superstring action under the $\kappa-$symmetry. In [17] we gave the proof that in generic case among 24 equations (18), (19) there are only 16 independent leading to the 8-parameter $\kappa-$symmetry of the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type action [14]. Such an 8-parameter $\kappa-$symmetry is the 'remnant' of 16-parameter symmetry of the full $AdS_4 \times CP^3$ superstring [18] remained upon gauging away 8 fermions related to space-time supersymmetries broken by the $AdS_4 \times CP^3$ superbackground. It should be noted that the above set of equations has to be supplemented by Virasoro conditions arising upon variation of (13) on auxiliary $2d$ metric g_{ij}:

$$\begin{aligned} &\tfrac{1}{4} (\omega_i^m + c_i^m)(\omega_{jm} + c_{jm}) + \Delta_i \Delta_j + \tfrac{1}{2} (\Omega_{ia} \Omega_j^a \\ &+ \Omega_{ja} \Omega_i^a) - \tfrac{1}{2} g_{ij} g^{i'j'} \left[\tfrac{1}{4} (\omega_{i'}^m + c_{i'}^m)(\omega_{j'm} + c_{j'm}) \right. \\ &+ \left. \Delta_{i'} \Delta_{j'} + \Omega_{i'a} \Omega_{j'}^a \right] = 0. \end{aligned} \qquad (20)$$

Equations of motion (15)-(19) can be obtained from the zero curvature condition

$$d\mathcal{L} - \mathcal{L} \wedge \mathcal{L} = 0 \qquad (21)$$

for the Lax connection 1-form $\mathcal{L}(d)$ taking value in the $osp(4|6)$ isometry algebra of the $AdS_4 \times CP^3$ superbackground [14]. Construction of the connection follows the same steps used to discover Lax representation for the $AdS_5 \times S^5$ superstring equations of motion [13]. In conformal basis for Cartan forms it can be written as

$$\mathcal{L} = \mathcal{L}_{conf_3} + \mathcal{L}_{su(4)} + \mathcal{L}_F, \qquad (22)$$

where

$$\begin{aligned} \mathcal{L}_{conf_3}(d) &= G^{mn} M_{mn} + \tfrac{1}{2} (c^m - \omega^m)(K_m - P_m) \\ &+ \tfrac{1}{2} [\ell_1 (\omega^m + c^m) + \ell_2{}^* (\omega^m + c^m)](P_m \\ &+ K_m) + (\ell_1 \Delta + \ell_2{}^* \Delta) D \in so(2,3) \end{aligned} \qquad (23)$$

and

$$\begin{aligned} \mathcal{L}_{su(4)}(d) &= \Omega_a{}^b V_b{}^a + \Omega_a{}^a V_b{}^b + (\ell_1 \Omega^a + \ell_2{}^* \Omega^a) V_a \\ &+ (\ell_1 \Omega_a + \ell_2{}^* \Omega_a) V^a \in su(4) \end{aligned} \qquad (24)$$

are Lax connections for bosonic string on AdS_4 and CP^3 manifolds respectively extended by the contributions of 24 anticommuting coordinates of $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset space. $2d$ Hodge dual of a 1-form $a(d)$ is defined as $^*a_i = \sqrt{-g}\varepsilon_{ij}g^{jk}a_k$. $\mathcal{L}_F$ is determined by the fermionic generators and Cartan forms:

$$\begin{aligned}\mathcal{L}_F(d) &= \ell_3 \tfrac{1}{2}(\omega^{\mu}_{a} + i\chi^{\mu}_{a})(Q^{a}_{\mu} + iS^{a}_{\mu}) \\ &+ \ell_4 \tfrac{1}{2}(\omega^{\mu}_{a} - i\chi^{\mu}_{a})(Q^{a}_{\mu} - iS^{a}_{\mu}) + \text{c.c.}\end{aligned} \quad (25)$$

Checking (21) requires using the Maurer-Cartan equations for Cartan forms (4) which explicit form is given in [17]. Also the zero curvature condition (21) restricts parameters ℓ_1,..., ℓ_4 to be functions of a single spectral parameter. One of their possible parametrizations is as follows:

$$\begin{aligned}\ell_1 &= \tfrac{1}{2}\left(z^2 + \frac{1}{z^2}\right), \quad \ell_2 = \tfrac{1}{2}\left(\frac{1}{z^2} - z^2\right), \\ \ell_3 &= z, \quad \ell_4 = \frac{1}{z},\end{aligned} \quad (26)$$

where z is assumed to be complex-valued non-zero. The Lax connection is defined modulo $OSp(4|6)$ gauge transformations $\mathcal{L}' = \mathrm{G}\mathcal{L}\mathrm{G}^{-1} - \mathrm{G}^{-1}d\mathrm{G}$. Special role is played by such a transformation with $\mathrm{G} = G$, where G is same as used to define Cartan forms in (4). Then the transformed Lax connection can be power series expanded in $w = -2\log z$ around zero:

$$\mathcal{L}' = w\,{}^*J + O(w^2), \quad (27)$$

where the leading contribution is given by the Hodge dual of $osp(4|6)$ superalgebra-valued Noether current density J associated with $OSp(4|6)$ global symmetry of the superstring action (13). Complete explicit form of the Noether currents corresponding to realization of the $OSp(4|6)$ global symmetry as $D = 3$ $\mathcal{N} = 6$ superconformal symmetry was obtained in [20].

To obtain explicit form of $OSp(4|6)/(SO(1,3) \times U(3))$ superstring action we have considered the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset element [17]

$$\begin{aligned}G &= \exp\left(x^m P_m + \theta^{\mu}_{a}Q^{a}_{\mu} + \bar\theta^{\mu a}\bar Q_{\mu a}\right)\exp\left(\eta_{\mu a}S^{\mu a}\right. \\ &+ \left.\bar\eta^{a}_{\mu}\bar S^{\mu}_{a}\right)\exp\left(z^a V_a + \bar z_a V^a\right)\exp\varphi D,\end{aligned} \quad (28)$$

parametrized by $D = 3$ $\mathcal{N} = 6$ super-Poincaré coordinates $(x^m, \theta^{\mu}_{a}, \bar\theta^{\mu a})$, AdS_4 radial direction coordinate φ, 3 complex coordinates $(z^a, \bar z_a)$ of the CP^3 manifold, and 12 anticommuting coordinates $(\eta_{\mu a}, \bar\eta^{a}_{\mu})$ associated with $D = 3$ $\mathcal{N} = 6$ conformal supersymmetry. Corresponding expressions for Cartan forms and superstring Lagrangian were derived in [17]. Similar choice of the supercoset representative was considered in [21] when studying the $AdS_5 \times S^5$ superstring in conformal basis for Cartan forms.

3. CONCLUSIONS

We have outlined Lagrangian formulation of $AdS_4 \times CP^3$ superstring as the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model. The procedure behind the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model construction [14, 15] is similar to that used previously to describe the $AdS_5 \times S^5$ superstring as the $PSU(2,2|4)/(SO(1,4) \times SO(5))$ sigma-model [9–13]. The main distinction is that the $osp(4|6)$ superalgebra has 24 supersymmetry generators in contrast to 32 generators of the $psu(2,2|4)$ superalgebra. It is traced back to the fact that $AdS_4 \times CP^3$ superbackground preserves 24 space-time supersymmetries, whereas the $AdS_5 \times S^5$ one preserves all 32 space-time supersymmetries. As a consequence the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type action describes the subsector of the $AdS_4 \times CP^3$ superstring [18] that does not take into account supersymmetries broken by the background. Important common feature of both supercoset sigma-models is that corresponding equations of motion are integrable and the Lax representation for them is known explicitly [13–15].

Aiming to elaborate a representation for the $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type Lagrangian that most of all fits symmetry structure of the dual gauge theory [7] we presented it in conformal basis for Cartan forms [17]. To this end we worked out the $osp(4|6)$ superalgebra realization as the $D = 3$ $\mathcal{N} = 6$ superconformal algebra. There was also given general derivation of the rank of matrices entering fermionic equations of motion and $\kappa-$symmetry transformations. Explicit expressions for the $osp(4|6)$ Cartan forms and $OSp(4|6)/(SO(1,3) \times U(3))$ sigma-model-type Lagrangian were found for the supercoset representative compatible with conformal structure. For the $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the Lagrangian complete expressions for the Noether currents were obtained [20]. This results hopefully will be of use in addressing such issues of ABJM correspondence as the spectrum identification, $T-$duality invariance, Wilson loops/scattering amplitudes duality.

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$AdS_4 \times CP^3$ СУПЕРСТРУНА КАК $OSp(4|6)/(SO(1,3) \times U(3))$ СИГМА-МОДЕЛЬ В КОНФОРМНОМ БАЗИСЕ

Д.В. Уваров

Обсуждается лагранжева динамика IIA суперструны в подпространстве $AdS_4 \times CP^3$ суперпространства, изоморфном $OSp(4|6)/(SO(1,3) \times U(3))$ фактор-многообразию. Ключевую роль при построении лагранжиана суперструны как классически-интегрируемой $OSp(4|6)/(SO(1,3) \times U(3))$ сигма-модели играет Z_4 дискретный автоморфизм $osp(4|6)$ супералгебры изометрии $AdS_4 \times CP^3$ суперпространства. Основное внимание уделяется представлению лагранжиана, следующих из него уравнений движения, а также связности Лакса через формы Картана для генераторов $D = 3\,\mathcal{N} = 6$ суперконформной алгебры.

$AdS_4 \times CP^3$ СУПЕРСТРУНА ЯК $OSp(4|6)/(SO(1,3) \times U(3))$ СИГМА-МОДЕЛЬ В КОНФОРМНОМУ БАЗИСІ

Д.В. Уваров

Обговорюється лагранжіва динаміка IIA суперструни в підпросторі $AdS_4 \times CP^3$ суперпростору, ізоморфному $OSp(4|6)/(SO(1,3) \times U(3))$ фактор-багатовиду. Ключову роль при побудові лагранжіана суперструни як класично-інтегровної $OSp(4|6)/(SO(1,3) \times U(3))$ сигма-моделі відіграє Z_4 дискретний автоморфізм $osp(4|6)$ супералгебри ізометрії $AdS_4 \times CP^3$ суперпростору. Основну увагу приділено зображенню лагранжіана, рівнянь руху, що з нього випливають, а також зв'язності Лакса через форми Картана для генераторів $D = 3\,\mathcal{N} = 6$ суперконформної алгебри.