NECESSARY GENERALIZATION OF KLEIN-GORDON AND DIRAC EQUATIONS AND EXISTENCE OF PARTICLE GENERATIONS

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It is shown that values of the Green functions for the Klein-Gordon and Dirac equations depend on calculation method. The integrals for the Green functions of new equations can converge if the minimal numbers of the particles equal three for spinless particles and five for 1/2-spin particles. It leads to the existence of massive generations for the photon and the gluons. It is shown that the interaction potentials have oscillatory form at short distances.

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1. INTRODUCTION

In the quark model it has been shown that the hadrons consist of the quarks of six flavors. Therefore now the leptons, the quarks, the photon, the gluons, the $W^{\pm}$, and the Z^0 only are consider as elementary particles. The study of the axial Adler-Bell-Jackiw anomaly has shown that the contribution of one 1/2-spin particle (a quark or a lepton) gives linear divergency. But taking into account of some sets of leptons and quark such as $e,\ \nu_e,\ u,\ d$ or $\mu,\ \nu_\mu,\ c,\ s$ or $\tau,\ \nu_\tau,\ t,\ b$ allows to eliminate this divergency. Thus the convergence of the axial anomaly gives the relation between the quarks and the leptons.

In connection with this the question arises: why do the generations of particles exist? We can remember the words of L.B. Okun that we good understand the reasons for the existence of some new particles. But we do not understand: why do old particles (for example the muon) exist? In present paper we show that for each particle must exist other particles with different masses but with the same spin, electric charge, and parity.

2. PARADOX OF GREEN FUNCTIONS

Consider the particle propagators, i.e. the Green functions. It is well known that in the static case the exchange by the particle of the mass m gives the Yukawa potential

$$V(r,\,m)=\frac{1}{4\pi}\frac{e^{-mr}}{r}.$$

This potential is the Green function

$$V\left(r,\,m\right)=G\left(\vec{x},m\right)=\frac{1}{\left(2\pi\right)^3}\int\frac{e^{i\vec{q}\vec{x}}}{\overrightarrow{q}^2+m^2}d^3q,\qquad(1)$$

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where $r=|\vec{x}|$. Note that we can put $m=0$ in Eqs. (1), for the Coulomb potential. In the relativistic case the exchange by the boson of the mass m can be expressed by means of the Green function for the Clein-Gordon-Fock equation

$$D(x,m)=\frac{1}{(2\pi)^4}\int\frac{e^{-iqx}d^4q}{-q^2+m^2}.\qquad(2)$$

For the 1/2-spin particle the Green function of the Dirac equation has a form

$$S(x,\,m)=\frac{1}{(2\pi)^4}\int\frac{(\widehat{q}+m)e^{-iqx}d^4q}{-q^2+m^2}.\qquad(3)$$

Usually the expressions for the Yukawa potential is derived from (1) by the calculations of the integrals in the spherical frame. Note that the integral in (1) is the infinite threefold integral. As it is known the improper (in particular infinite) integral converges in that case only if the calculations of it give the same finite result by any possible methods. The convergence of improper onefold and multiple integrals have some distinctions. Such for multiple improper integral the conditional convergence does not exist. In [1, 2] it is proved that if the twofold improper integral converges then it converges absolutely also (i.e. the improper twofold integral with the module of the integrand converges). This is valid for any multiple improper integral too [2]. Thus for the multiple improper integrals the convergence and the absolute convergence are equivalent [2]. Therefore multiple improper integral converges then and only then when this integral converges absolutely. Thus the integral in (1) converges only in case of the convergence of the integral

$$\frac{1}{(2\pi)^3}\int\frac{d^3q}{\overrightarrow{q}^2+m^2}.\qquad(4)$$

But this integral diverges. Therefore the integral in (1) diverges. To see this we shall integrate (1) in the cylindrical frame. We choose $\overrightarrow{x} = (0, 0, r)$. Then $\vec{q}\vec{x} = q_3 r$ and $d^3q = 1/2d\,|\vec{q}_\perp|^2\, d\varphi dq_3$. We shall integrate in the next order: with respect to the angle φ, $|\vec{q}_\perp|$, q_3 , respectively. Thus we derive

$$G(\vec{x},\, m) = \frac{1}{8\pi^2}\int\limits_{-\infty}^{+\infty} e^{iq_3 r} dq_3 \int\limits_{0}^{\infty} \frac{d\vec{q}_\perp^{\,2}}{\overrightarrow{q}_\perp^{\,2} + q_3^2 + m^2} =$$

$$= \frac{1}{8\pi^2}\int\limits_{-\infty}^{+\infty} e^{iq_3 r} dq_3 \left[\lim_{\vec{q}_\perp^{\,2}\to\infty} \ln\left(\overrightarrow{q}_\perp^{\,2} + q^3 + m^2\right) - \right.$$

$$\left. - \ln(q_3^2 + m^2)\right] = \frac{1}{2\pi}\delta(r) \lim_{|\overrightarrow{q}_\perp|\to\infty} \ln|\overrightarrow{q}_\perp| -$$

$$- \frac{1}{2\pi^2\, r} \lim_{q_3\to\infty} \ln q_3 \sin q_3 r + \frac{1}{4\pi}\cdot\frac{e^{-mr}}{r}. \quad (5)$$

We see that this integral diverges as the first term is indefinite and the limit in the second term does not exist, but these diverging terms do not depend on the particle mass m.

Thus we derive the paradox (paradox of the Green functions). **From the mathematical point of view the use of the Green functions (1)-(3) is incorrect, but these Green functions (calculated by some fashion) give adequate description of different experimental data.**

In Ref. [3] it is shown that the integrals (2), (3) diverge too. We may assume that the solution of the Green function paradox is possible by two ways: 1) we can conclude that existing theory is wrong and we must find new theoretical approach based on new mathematical methods; 2) we can try to modify existing theory.

3. GENERALIZATIONS OF KLEIN-GORDON AND DIRAC EQUATIONS

We consider second way by means of proper modification of the Green functions and corresponding generalization of the Klein-Gordon and Dirac equations. We propose: 1) the generalizations of the Klein-Gordon and Dirac equations must have some simple form; 2) the existing expressions can be derived from new generalized Green functions in some limit. We propose that the generalized non-homogeneous Klein-Gordon equation is the $2N$-order equation and may be written as

$$(\Box + m_1^2)(\Box + m_2^2)...(\Box + m_N^2)\varphi(x) = \eta(x), \quad (6)$$

where $\varphi(x)$ is the field and $\eta(x)$ is the current (the field source). The Green function for Eq. (6) is given by

$$\bar{G}(x) = \frac{1}{(2\pi)^4}\int \frac{e^{-iqx} d^4q}{P_N(q^2)} = \frac{1}{(2\pi)^4}\cdot$$

$$\cdot\int \frac{e^{-iqx} d^4q}{(-q^2 + m_1^2)(-q^2 + m_2^2)...\,(-q^2 + m_N^2)}, \quad (7)$$

where $P_N(q^2)$ is the polynomial of the N degree with respect to q^2.

The general classical solution $\varphi_{cl}(x)$ of the linear equation (6) is the sum of the general solution of the corresponding homogeneous equation $\varphi_f(x)$ and partial solution $\varphi_{nh}(x)$ of non-homogeneous equation:

$$\varphi_f(x) = \int d^4q \sum_{k=1}^{N} \delta(q^2 - m_k^2)\left[c_k e^{-iqx} + \tilde{c}_k e^{iqx}\right], \quad (8)$$

where c_k and $\tilde{c}_k$ are the arbitrary constants. Thus $\varphi_f(x)$ is the sum on the terms corresponding to particles with the same charges, parity, spin but with different masses. Each term in (8) corresponding to number k is the solution of the homogeneous Klein-Gordon equation. In Ref. [3] it is shown, that the functions $\varphi_f(x)$ are non-normalizable if at least two masses are equal. Thus the masses in the generalized Klein-Gordon equation must be different. We can write

$$\frac{1}{(-q^2 + m_1^2)(-q^2 + m_2^2)...(-q^2 + m_N^2)} =$$

$$= \frac{1}{P_N(q^2)} = \sum_{k=1}^{N} \frac{A_k}{-q^2 + m_k^2} =,$$

$$A_k = -\frac{1}{P'_N(m_k^2)} = \lim_{q^2\to m_k^2} \frac{-q^2 + m_k^2}{P_N(q^2)},$$

$$A_k = (-1)^{k+1}|A_k|. \quad (9)$$

For the A_k coefficients the relations are valid:

$$\sum_{k=1}^{N} A_k m_k^{2l} = 0, \quad l = 0,\, 1,\, 2,\, ...,\, N-2, \quad (10)$$

$$\sum_{k=1}^{N} A_k m_k^{2N-2} = 1. \quad (11)$$

Using the equality (9) we can express the Green function (7) of Eq. (6) in terms of the Green functions (3)

$$\bar{G}(x) = \sum_{k=1}^{N} A_k D(x,\, m_k). \quad (12)$$

As the dimension of the time-space is equal to four the integral (7) can be convergent at $N \geq 3$. Consequently for each spinless particle two (or greater) particles with the same charges, C- and P-parity, but different masses, must exist in addition. We may say that such particles are members of some set (a family or a kind or a dynasty). The members of different kinds belong to the generation. In Eqs. (8), k is the number of the particle generation. We may assume that the member quantity for the elementary particle kinds are less than the member quantity for the composite particle kinds. Each particle belongs to some kind and some generation.

For the 1/2-spin particles we propose the next generalization of the non-homogeneous Dirac equation

$$(m_1 - i\widehat{\partial})(m_2 - i\widehat{\partial})...(m_N - i\widehat{\partial})\psi(x)^\alpha = \chi(x)^\alpha, \quad (13)$$

where α is the bispinor index. The Green function for this equation may be written as

$$\bar{S}(x) = \frac{1}{(2\pi)^4}\times$$

$$\times \int \frac{(\widehat{q}+m_1)(\widehat{q}+m_2)....(\widehat{q}+m_N)}{(-q^2+m_1^2)(-q^2+m_2^2)...(-q^2+m_N^2)} d^4 q. \quad (14)$$

The integral (14) can be convergent at $N \geq 5$ only. Thus for each $\frac{1}{2}$-spin particle four (or greater) particles with the same charges, isospin, P-parity, but with different masses, must exist in addition. In Ref. [3] the formulae for the $\frac{1}{2}$-spin particles similar to formulae (9)-(12) are derived.

4. ABSENCE OF SINGULARITIES IN GREEN FUNCTIONS OF GENERALIZED KLEIN-GORDON AND DIRAC EQUATIONS

Since the generalized Klein-Gordon equation (6) and generalized Dirac equation (13) have degree greater than four their Green functions and their first partial derivatives can be continuous function of the time and spatial variables, i.e. these Green functions cannot have any singularities (more precisely these Green functions can have the removable discontinuity). Note that the Green functions of the Klein-Gordon equation have singularities on the light cone, such as $\delta(x^2),\ 1/x^2,\ \Theta(x^2),\ \ln|x^2|$ [4, 5]. The singularities disappear in causal $\overline{D}(x)_c$, advanced $\overline{D}(x)_{adv}$, and retarded $\overline{D}(x)_{ret}$ by similar fashion. For example, we have the generalization of the Yukawa potential

$$\bar{G}(\overrightarrow{x}) = \sum_{k=1}^{N} A_k G(\overrightarrow{x},\, m_k) = \frac{1}{4\pi}\sum_{k=1}^{N} A_k \frac{e^{-m_k r}}{r}. \quad (15)$$

Each term of the sum in (15) has singularity at $r = |\overrightarrow{x}| = 0$ (i.e. on the light cone $x^2 = 0 - r^2 = 0$). Using the expansion $e^{-m_k r} = 1 - m_k r + \frac{m_k^2 r^2}{2} - \frac{m_k^3 r^3}{6} + ...$ at small r and relations (10) for $l = 0$ and 1 we derive

$$\bar{G}(\overrightarrow{x}) = -\frac{1}{4\pi}\sum_{k=1}^{N} A_k (m_k + \frac{m_k^3 r^2}{6}). \quad (16)$$

This $\bar{G}(\overrightarrow{x})$ has no any singularities, as contrast with the Goulomb and Yukawa potentials. From (16) we see that at short distances the potential must have the form of harmonic oscillator. The oscillatory potentials are widely used in the nuclear physics and in the quark models. The interaction force at small r

$$\overrightarrow{F}(\overrightarrow{x}) = -\mathrm{grad}\,\overline{G}(\overrightarrow{x}) = \frac{\overrightarrow{x}}{12\pi}\sum_{k=1}^{N} A_k m_k^3, \quad (17)$$

has no any singularities too. It is interesting to note that $\vec{F}(0) = 0$. Therefore we may assume that all the interactions must be relaxed at short distances. It is similar to asymptotic freedom.

Note that if to use for $V(r, m_k)$ the result (5) derived in the cylindrical frame then the contributions of the diverging terms vanish, as consequence of the relation (11) at $l = 0$. Thus we derive (16), (17) for $G(\overrightarrow{x}, m_k)$ (1) and (5). This confirms the convergence of $\overline{G}(\overrightarrow{x})$.

5. SOLUTION OF GREEN FUNCTION PARADOX

Consider the question about the reproduction of the results derived early (such as Yukawa potential) in our approach. It easy to see from (15) that at relatively large r in the sum the term including m_1 is important only, i.e. at relatively large r $\bar{G}(\vec{x})$ approximately is equal to the Yukawa potential. Simultaneously large r corresponds to small components of the q-momentum. Assume that $m_1/m_k \ll 1$ for $k = 2, 3, ..., N$. Then we can rewrite approximately the equations (6) and (13) in forms

$$(\Box + m_1^2) m_2^2 ... m_N^2 \varphi(x) = \eta(x),$$
$$(-i\,\widehat{\partial} + m_1) m_2 ... m_N \psi(x) = \chi(x). \quad (18)$$

These equations practically coincide with the non-homogeneous Klein-Gordon and Dirac equations for the particles with the m_1 mass. We can reduce at large distances (i.e. in low-energy approximation) the equations (6) and (13) to the non-homogeneous Klein-Gordon and Dirac equations, respectively, by means of the redefinitions of the interaction currents. We have seen from (5) that the calculations of the Coulomb and Yukawa potentials by means of the integral (1) are incorrect. But we have derived these potentials as large-distance limit of the Green function for the generalization of the Klein-Gordon equation in the static case (15). In consequence of this and approximate validity of the Klein-Gordon equation at low energies (at large distances) we may assume that the use of the Coulomb and Yukawa potentials in the low-energy physics is admissible. In particular the results derived in the solid state physics, the plasma physics, the statistical physics, the atomic physics, and low-energy nuclear physics are valid.

6. KINDS OF ELEMENTARY PARTICLES

Consider the distribution of the elementary particles in the kinds (or the dynasties). For the photon and gluon $m_1 = 0$. Since for the particle of integer spin $N \geq 3$ two (or greater) massive members of the photon kind must exist. They must have zero electric charge, $J^p = 1^-$, $C = -1$. These particle must contribute to amplitudes of $e^+e^- \to e^+e^-$, $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to$ hadrons at high energies and give the resonance behavior. We can expect that the coupling constants for the interactions of these members of the photonic kind with the leptons and the hadrons of the same electric charges must be equal. Therefore the vector mesons $\rho,\ \omega,\ \varphi,\ J/\psi$ cannot be the members of the photonic kind. We conclude that the corrections to the Coulomb law must be at small distance in addition to the corrections of the quantum electrodynamics [6]. Similarly in the gluonic kind two (or greater) massive colored particles must exist. Besides two (or greater) massive members must exist in the Z^0- and $W^{\pm}$-kinds. In relations with the necessary existence of massive photons and gluons such questions arise: 1) Is the gauge invariance for massive photons and gluons possible or not? 2) Does the scaling in deep inelastic lepton – hadron scattering at higher energies exist or not?

It has been shown that for the $\frac{1}{2}$-spin particles the number of the kind members (i.e. generations) must be equal to 5 (or greater). We assume that electron kind ($e_1 = e,\ e_2 = \mu,\ e_3 = \tau,\ e_4,\ e_5,\ \ldots$), the neutrino kind ($\nu_1 = \nu_e,\ \nu_2 = \nu_\mu,\ \nu_3 = \nu_\tau,\ \nu_4,\ \nu_5, ...$), three kinds of the coloured *up*-quarks ($u_1 = u,\ u_2 = c,\ u_3 = t,\ u_4\ , u_5, ...$), and three kinds of the coloured *down*-quarks ($d_1 = d,\ d_2 = s,\ d_3 = b,\ d_4,\ d_5, ...$) exist. Note that in our approach only one neutrino may be massless. The higher members of the electron and quark kinds can decay. For example e_4 and e_5 can decay into $e\,\nu\,\bar{\nu},\ \ \mu\,\nu\,\bar{\nu}$ (similarly to $\mu \rightarrow e\,\nu\,\bar{\nu}$), and $\nu+$ hadrons. We can assume the possibility of radiative decays $e_4,\ e_5 \rightarrow \mu\,\gamma$ or $e_4, e_5 \rightarrow \mu\,\gamma\,\gamma$. We can expect that such interactions of the higher Z^0 and $W^{\pm}$ will be fairly weak in comparison with the interactions of $Z_1^0 = Z^0(92.4)$ and $W_1^{\pm} = W^{\pm}(81)$ (in GeV), as consequence of big masses of the higher Z^0 and $W^{\pm}$. We may assume that: 1) Z_2^0 or Z_3^0 or $W_2^{\pm}$ or $W_3^{\pm}$ can interact with right currents; 2) the interactions of Z_2^0 or Z_3^0 with fermion may be determined by the mixing matrix similar to the Kobayashi-Maskawa matrix and Z_2^0 or Z_3^0 can induce the transitions between the fermions of different generations (like to the $s \rightarrow W_1^- u$-transition). Therefore in addition to the investigations of the decays $Z_{2,\,3}^0 \rightarrow \mu^+\mu^-$X [7] it is of interest the study of the decays $Z_{2,\,3}^0 \rightarrow \mu^{\pm}e^{\mp}$X, which are forbidden in the Standard Model.

If higher neutrino are enough heavy then fairly exotic decay $\nu_{4,\,5} \rightarrow e\mu\nu_{1,\,2}$ becomes possible. Since for fermions $N \geq 5$ the Kobayashi-Maskawa matrix must have the fifth (or greater) order. This can be important for the effects of CP-violation.Possibly the leptons and the quarks from the fourth and fifth generations can be observed in Fermilab or LHC.

7. ON EXISTENCE OF BLACK HOLES

As it is known the assumed black holes have high density and induce such strong gravitational fields that any particles including photons, cannot be emitted. In accordance with the calculations the density of the black holes can be higher than the nuclear density. Possibly the black holes correspond to the singularities in the time-space. But the existence of the black holes are predicted in the classical physics without the considerations of the quantum effects, which are just important on small distances. It is known that the classical physics leads to the contradictions with the reality on atomic distances, i.e. on atomic radii and distances between the atoms in the water and solid states (which correspond to the water density). Indeed in accordance with the classical physics the electron must lose the energy in consequence of the light emission and it must fall on a nucleus. Thus in accordance with a classical physics an atom cannot exist more than 10^{-10} c. The stability of atoms is derived just in the quantum mechanics, i.e. taking into account of quantum effects allows to understand the phenomena at short distances correctly. The distances corresponding to black holes are less than 1 fm. Thus we may conclude that the question on the existence of black holes can be solved in the quantum theory of the gravitation only.

Consider some quantum effects. 1) It is well known that the stable state of any physical system corresponds to the lowest energy level. The object before the transition into the black hole has a strong gravitational field and this object can reduce own energy level by means of the photon radiation and in consequence of this the strength of the gravitational field will decrease. Thus such object will not transit into black hole. This agrees with the result derived by Howking that the small black holes disappear; 2) Consider the elastic electron – proton scattering in the one-photon (or one-graviton) exchange approximation. In c.m.s. the transfer energy equals zero, but the 3-momentum transfer is nonzero, i.e. the 4-momentum of the virtual space-like photon $q = (0,\ \overrightarrow{q})$, where $\overrightarrow{q} \neq 0$. It corresponds to the infinite velocity of the virtual space-like photon. Possibly it may be interpreted that the velocities of virtual particles can be superlight on small distances; 3) In Ref. [3] the equations for 1- and 2-spin fields with $m_1 = 0$ are derived. These equations have form of Eq. (6). Therefore the solutions of these equations must be continuous functions and the force of the gravitational interaction is proportional to the distance, in agreement with Eq. (16) on small distances.

We may assume that in addition to the massless graviton must exist two (or greater) massive gravitons, similarly to the photons and the gluons. Note that the since the gravitons have got the spin 2 the currents of the gravitation interaction must obey the theorem on currents and fields as well as the theorem on current asymptotics [8-12]. Possibly in such approach the anomaly related to the interaction of the graviton with two photons will be finite, i.e. the quadratic divergence [13] disappears.

Thus the consideration of the quantum effects allows to expect that the existence of black holes is very problematic.

8. CONCLUSION

We have shown that the Klein-Gordon and Dirac equations must be modified as the integrals for their Green functions diverge. The integrals for Green functions of proposed equations for 0- and 1/2-spin particles can converge only at existence of new high-mass particles. New Green function have no singularities in the space-time.

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НЕОБХОДИМОЕ ОБОБЩЕНИЕ УРАВНЕНИЙ КЛЕЙНА-ГОРДОНА И ДИРАКА И СУЩЕСТВОВАНИЕ ПОКОЛЕНИЙ ЧАСТИЦ

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Показано, что значения функций Грина уравнений Клейна-Гордона и Дирака зависят от метода вычисления. Интегралы для функций Грина новых уравнений могут сходиться, если минимальное количество частиц равно трем для бесспиновых частиц и пять для частиц со спином 1/2. Это приводит к существованию массивных поколений для фотонов и глюонов. Показано, что потенциалы взаимодействия на малых расстояниях имеют осцилляторную форму.

НЕОБХІДНЕ УЗАГАЛЬНЕННЯ РІВНЯНЬ КЛЕЙНА-ГОРДОНА І ДІРАКА ТА ІСНУВАННЯ ПОКОЛІНЬ ЧАСТИНОК

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Показано, що значення функцій Гріна рівнянь Клейна-Гордона та Дірака залежать від метода обчислень. Інтеграли для функцій Гріна нових рівнянь можуть збігатися, якщо мінімальна кількість частинок дорівнює трьом для безспінових частинок і п'яти для частинок із спіном 1/2. Це приводить до існування масивних поколінь для фотонів і глюонів. Показано, що потенціали взаємодій на малих відстанях мають осциляторну форму.