РАСПРОСТРАНЕНИЕ И РАССЕЯНИЕ ВОЛН

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WAVE TRANSFORMATION FROM STATISTICALLY ROUGH SURFACE

F. G. Bass, L. B. Vatova

Памяти С. Я. Брауде, инициировавшего исследования в данном направлении

Department of Physics, Bar-Ilan University, Ramat-Gan 52900, Israel
E-mail: bassfr@mail.biu.ac.il

The generalized small perturbation method is applied to investigate the transformation of waves on statistically rough surface separating media in which several types of waves caused by a time and spatial dispersion propagate. The field boundary conditions have been formulated. The mean intensity of the transformed fields have been calculated.

Key words: statistically rough surface, time and spatial dispersion, wave transformation, small perturbation method, saddle-point technique.

Wave scattering from statistically rough surface is a subject of current interest [1, 2]. Transformation and scattering of waves from a randomly rough interface between media, in which several types of waves propagate, have been studied in the Kirchhoff approximation [3]. Wave scattering is the particular case of transformation when an incident wave transforms to a wave of the same type. Spatial and time dispersion, of media give rise to the occurrence of several waves which differ from each other by their dispersion law, i. e., the frequency dependence on the wave vector.

In the present work the method of small perturbations is generalized and used to investigate the transformation of waves in cases in which the Kirchhoff approximation can not be applied.

Let us consider two semi-infinite half spaces of different media with several type of waves. For the sake of simplicity, we assume that the number of waves is the same in each medium. The media are separated from each other by a statistically rough surface

$$z = \varsigma(x, y), \tag{1}$$

where $\varsigma(x, y)$ is a random uniform function with the following statistical properties:

$$\langle \varsigma(x, y) \rangle = 0; \tag{2, a}$$

$$\langle \varsigma(x, y)\varsigma(x', y') \rangle = K(x - x', y - y'); \tag{2, b}$$

$$\langle \varsigma^2 \rangle = K(0) = \text{const}, \tag{2, c}$$

where $K(x - x', y - y')$ is the correlation function, and averaging is denoted by brackets $\langle ... \rangle$. As one can see from Eqs. (2, a – 2, c), the mean value of the function ς is equal to zero, and the correlation function K is a function of the differences $x - x'$, $y - y'$. The applicability test of the small perturbation method [1, 2] is supposed to be carried out, i. e.

$$\left| \frac{\varsigma(x, y)}{\Lambda} \right| << 1, \quad \left| \frac{\partial \varsigma}{\partial x} \right|, \left| \frac{\partial \varsigma}{\partial y} \right| << 1, \tag{3}$$

where Λ is the smallest length of the waves propagating in both media. In what follows, in the medium $z > \varsigma(x, y)$ all variables are denoted by index α while in $z < \varsigma(x, y)$ by index β.

The equations describing the field E_α and E_β can be presented as follows:

$$H_{\alpha,\beta}\left(-\frac{1}{i}\frac{\partial}{\partial t}, \frac{1}{i}\frac{\partial}{\partial \tilde{\vec{r}}} \right) E_{\alpha,\beta}\left(\tilde{\vec{r}}, t\right) = 0, \tag{4}$$

where H_α and H_β are arbitrary functions of their arguments $-\frac{1}{i}\frac{\partial}{\partial t}$, $-\frac{1}{i}\frac{\partial}{\partial \tilde{\vec{r}}}$. Eqs. (4) can be partial differential equations, integral equations with difference kernels, finite difference equations, and equations of mixed type, describing a wide range of wave processes [4].

The equations must be supplemented with boundary conditions on the surface (1) and radiation conditions. As Eqs. (4) have a general form, the form of the boundary conditions should be the same. The similar approach is applied in [5]. The boundary conditions on the surface (1) can be written in the following general form

$$\begin{aligned} &Q_{\alpha m}\left(\varsigma + 0, \frac{1}{i}\frac{\partial}{\partial n} \right) E_\alpha\left(\tilde{\vec{r}}_\perp, \varsigma + 0\right) = \\ &= Q_{\beta m}\left(\varsigma - 0, \frac{1}{i}\frac{\partial}{\partial n} \right) E_\alpha\left(\tilde{\vec{r}}_\perp, \varsigma - 0\right), \end{aligned} \tag{5}$$

where $\tilde{\vec{r}}_\perp$ is a vector with components (x, y);

$$\frac{\partial}{\partial n} = \frac{\dfrac{\partial}{\partial z} - \dfrac{\partial \varsigma}{\partial \tilde{\vec{r}}_\perp}\dfrac{\partial}{\partial \tilde{\vec{r}}_\perp}}{\sqrt{1 + \left(\dfrac{\partial \varsigma}{\partial \tilde{\vec{r}}_\perp} \right)^2}}$$

is the derivative with respect to the normal; $m = 1,2,...L$; L is the number of waves in the media α and β.

The operators $Q_{\alpha,\beta}$ in the boundary conditions (5) can be differential, integral, finite difference and mixed type.

If the inequalities (3) are satisfied, the fields E_α and E_β can be found in the form

$$E_\alpha = E_{\alpha 0} + E_{\alpha 1};\; E_\beta = E_{\beta 0} + E_{\beta 1}; \qquad (6, a)$$

and

$$\left|E_{\alpha 0}\right| >> \left|E_{\alpha 1}\right|;\; \left|E_{\beta 0}\right| >> \left|E_{\beta 1}\right|. \qquad (6, b)$$

The variables $E_{\alpha 0}$ and $E_{\beta 0}$ describe the interaction of waves with the unperturbed interface $z = 0$ while $E_{\alpha 1}$ and $E_{\beta 1}$ characterize the scattering on the roughness $\varsigma(x, y)$. In this approximation, the square root in the expression for $\frac{\partial}{\partial n}$ can be replaced by unity. In the approximation of the small perturbation method, $Q_{\alpha,\beta,m}$ can be expanded as a power series in ς and $\frac{\partial\varsigma}{\partial r}$

$$\begin{gathered} Q_{\alpha m}\left(\varsigma, \frac{\partial}{\partial z} - \frac{\partial\varsigma}{\partial\tilde{\vec r}_\perp}\frac{\partial}{\partial\tilde{\vec r}_\perp}\right) = Q^{(0)}_{\alpha m}\left(0, \frac{1}{i}\frac{\partial}{\partial z}\right) + \\ + Q^{(1)}_{\alpha m}\left(0, \frac{1}{i}\frac{\partial}{\partial z}\right)\varsigma - Q^{(2)}_{\alpha m}\left(0, \frac{1}{i}\frac{\partial}{\partial z}\right)\frac{\partial\varsigma}{\partial\tilde{\vec r}_\perp}\frac{\partial}{\partial\tilde{\vec r}_\perp}, \end{gathered} \qquad (7)$$

where

$$Q^{(0)}_{\alpha m}\left(\frac{1}{i}\frac{\partial}{\partial z}\right) = Q_{\alpha m}\left(0, \frac{1}{i}\frac{\partial}{\partial z}\right);$$

$$Q^{(1)}_{\alpha m}\left(\frac{1}{i}\frac{\partial}{\partial z}\right) = \frac{\partial Q_{\alpha m}\left(0, \frac{1}{i}\frac{\partial}{\partial z}\right)}{\partial\varsigma};$$

$$Q^{(2)}_{\alpha m}\left(\frac{1}{i}\frac{\partial}{\partial z}\right) = \frac{\partial Q_{\alpha m}\left(0, \frac{1}{i}\frac{\partial}{\partial z}\right)}{\partial\frac{1}{i}\frac{\partial}{\partial z}}.$$

The replacement of α by β in Eq. (7) gives rise to the expansion for $Q_{\beta m}$. The boundary conditions for the fields of zero and first approximations can be obtained from Eqs. (7). The result is

$$Q^{(0)}_{m\alpha}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)E_{\alpha 0} - Q^{(0)}_{m\beta}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)E_{\beta 0} = 0; \qquad (8)$$

$$\begin{gathered} Q^{(0)}_{m\alpha}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)E_{\alpha 1} - Q^{(0)}_{m\beta}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)E_{\beta 1} = \\ = \left[Q^{(1)}_{m\beta}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)E_{\beta 0} - Q^{(1)}_{m\alpha}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)E_{\alpha 0}\right]\varsigma - \\ - \left[Q^{(2)}_{m\beta}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)\frac{\partial E_{\beta 0}}{\partial\tilde{\vec r}_\perp} - Q^{(2)}_{m\alpha}\left(\frac{1}{i}\frac{\partial}{\partial z}\right)\frac{\partial E_{\alpha 0}}{\partial\tilde{\vec r}_\perp}\right]\frac{\partial\varsigma}{\partial\tilde{\vec r}_\perp}. \end{gathered} \qquad (9)$$

As noted above, the boundary conditions (5) are given in the general form and can be used to solve a wide range of problems. In Eq. (5), Q depends on $\frac{\partial}{\partial\tilde{\vec r}_\perp}$, however, this dependence does not change the essence of the matter and may be omitted. The boundary conditions (8) and (9) should be supplemented with radiation conditions. The fields of zero – $E_{\alpha 0}$, $E_{\beta 0}$ and first approximations – $E_{\alpha 1}$, $E_{\beta 1}$ satisfy the equations (4).

Let us assume that the fields are monochromatic and have the following form

$$E_{\alpha,\beta}\left(\tilde{\vec r}, t\right) = E_{\alpha,\beta}(\omega, \vec r)\exp(-i\omega t).$$

Substituting the expressions for $E_{\alpha,\beta}\left(\tilde{\vec r}, t\right)$ into Eqs. (4), the equations for $E_{\alpha,\beta}(\omega, \vec r)$ can be obtained

$$H_{\alpha,\beta}\left(\omega, \frac{1}{i}\frac{\partial}{\partial\tilde{\vec r}}\right)E_{\alpha,\beta}\left(\tilde{\vec r}, \omega\right) = 0, \qquad (10)$$

where

$$E_{\alpha,\beta}(\omega, \vec r) = E_{\alpha 0,\beta 0}(\omega, \vec r) + E_{\alpha 1,\beta 1}(\omega, \vec r),$$

and $E_{\alpha 0,\beta 0}(\omega, \vec r)$, $E_{\alpha,\beta}(\omega, \vec r)$ satisfy the Eq. (10). In what follows, the argument ω will be omitted if no misunderstandings follow.

Firstly, the problem of zero approximation, i. e. Eqs. (10) with the boundary conditions (9) and the radiation conditions is solved. Let us suppose that L different waves propagate in the media α and β. Then, the solution of Eqs. (10) has the form

$$\begin{gathered} E_{\alpha 0}\left(\tilde{\vec r}\right) = \\ = \sum_{l=1}^{L} A^{(0)}_l\left(\tilde{\vec k}_{\perp l}\right)\exp\left\{i\left[\tilde{\vec k}_{\perp l}\tilde{\vec r}_\perp - k_{zl}\left(\tilde{\vec k}_{\perp l}\right)z\right]\right\} + \\ + \sum_{l=1}^{L} A_{l0}\left(\tilde{\vec k}_{\perp l}\right)\exp\left\{i\left[\tilde{\vec k}_{\perp l}\tilde{\vec r}_\perp + k_{zl}\left(\tilde{\vec k}_{\perp l}\right)z\right]\right\}; \end{gathered} \qquad (11)$$

$$E_{\beta 0}\left(\tilde{\vec r}\right) = \sum_{l=1}^{L} B_{l0}\left(\tilde{\kappa}_{\perp l}\right)\exp\left\{i\left[\tilde{\kappa}_{\perp l}\tilde{\vec r}_\perp - \kappa_{zl}\left(\tilde{\kappa}_{\perp l}\right)z\right]\right\}, \qquad (12)$$

where $\vec k_\perp$, $\vec\kappa_\perp$, $\vec r_\perp$ are vectors with components k_x, k_y; κ_x, κ_y; x, y correspondingly.

The first term in Eq. (11) describes the incident waves, the coefficients $A^{(0)}_l\left(\tilde{\vec k}_{\perp l}\right)$ are known. Each of the incident waves is transformed to a complete set of waves which can propagate in the media. Eq. (12) describes the waves penetrating into the medium β. The coefficients A_{l0} and B_{l0} are obtained from the boundary conditions (9). Substitution of Eqs. (11) and (12) into Eqs. (10) gives rise to the dispersion equation for finding $k_{zl}\left(\tilde{\vec k}_{\perp l}\right)$ and $\kappa_{zl}\left(\tilde{\kappa}_{\perp l}\right)$.

$$H_\alpha\left(\tilde{\vec{k}}_{\perp l}, k_{zl}\right) = 0;\ H_\beta\left(\tilde{\kappa}_{\perp l}, \kappa_{zl}\right) = 0. \qquad (13)$$

As L waves propagate in both media, each equation in Eq. (13) has L real roots k_{zl} and κ_{zl} (l is the number of root).

On substituting Eq. (11) and Eq. (12) into the boundary conditions (9), one can obtain the following results for $\vec{k}_{\perp l}$ and $\vec{\kappa}_{\perp l}$

$$\tilde{\vec{k}}_{\perp l} = \tilde{\vec{k}}_{\perp l_1} = \tilde{\kappa}_{\perp l_2} = \tilde{\vec{k}}_{\perp}, \qquad (14)$$

where l, l_1, l_2, l_3 are integer from 1 to L. Eq. (14) can be consider as the Snellius law. The equations for finding A_{l0} and B_{l0} are as follows:

$$\sum_{m'=1}^{2L} S_{mm'}\eta_{m'} = \xi_m, \qquad (15)$$

where

$$S_{mm'} = \begin{cases} Q_{\alpha m}\left(k_{zm'}\right) \text{at } m = 1...L, m' = 1...L; \\ -Q_{\beta m}\left(-\kappa_{m'-L}\right) \text{at } m = 1...L, m' = L+1...2L; \end{cases}$$

$$\xi_m = \sum_{m=1}^{2L} Q_{\alpha m}\left(-k_{zm}\right) A_l^{(0)};$$

$$\eta_{m'} = \begin{cases} A_{m'0}\left(\tilde{\vec{k}}_\perp\right) \text{at } m' = 1...L; \\ B_{(m'-L)0}\left(\tilde{\vec{k}}_\perp\right) \text{at } m' = L+1...2L; \end{cases}$$

$$\xi_m = \sum_{m'=1}^{L} Q_{\alpha m}\left(-k_{zm'}\right) A_{m'}^{(0)}.$$

The solution of Eq. (15) is

$$\eta_m = \frac{1}{D}\sum_{m'=1}^{2L} D_{mm'}\xi_{m'}, \qquad (16)$$

where D and $D_{mm'}$ are the determinant and cofactor of the set of Eqs. (15).

Calculation of the fields $E_{\alpha 0}$ and $E_{\beta 0}$ results

$$\begin{aligned} E_{\alpha 0} = \exp\left[i\left(\tilde{\vec{k}}_\perp \tilde{\vec{r}}_\perp\right)\right]&\left\{\sum_{l=1}^{L} A_l^{(0)}\left(\tilde{\vec{k}}_\perp\right)\times\right. \\ \times\exp\left[-ik_{zl}\left(\tilde{\vec{k}}_\perp\right)z\right] &+ \\ + \sum_{l=1}^{L} A_{l0}\left(\tilde{\vec{k}}_\perp\right)&\left.\exp\left[ik_{zl}\left(\tilde{\vec{k}}_\perp\right)z\right]\right\}; \end{aligned} \qquad (17)$$

$$\begin{aligned} E_{\beta 0} &= \exp\left[i\left(\tilde{\kappa}_\perp \tilde{\vec{r}}_\perp\right)\right]\sum_{l=1}^{L} B_{l0}\left(\tilde{\kappa}_\perp\right)\times \\ &\times\exp\left[-i\kappa_{zl}\left(\tilde{\kappa}_\perp\right)z\right]. \end{aligned} \qquad (18)$$

Note that any incident wave of one type transforms to waves of all types. The fields of the second approximation $E_{\alpha 1}$ and $E_{\beta 1}$ describing the waves scattered from statistical roughness occupying the surface S, can be found.

These fields must satisfy Eqs. (10), the radiation conditions, and the boundary condition (9). They have the form

$$\begin{aligned} E_{\alpha 1}\left(\tilde{\vec{r}}\right) &= \sum_{l=1}^{L}\int A_{l1}\left(\tilde{\vec{q}}_\perp\right)\times \\ &\times\exp\left\{i\left[\tilde{\vec{q}}_\perp \tilde{\vec{r}}_\perp + k_{zl}\left(\tilde{\vec{q}}_\perp\right)z\right]\right\}d\tilde{\vec{q}}_\perp; \end{aligned} \qquad (19)$$

$$\begin{aligned} E_{\beta 1}\left(\tilde{\vec{r}}\right) &= \sum_{l=1}^{L}\int B_{l1}\left(\tilde{\vec{q}}_\perp\right)\times \\ &\times\exp\left\{i\left[\tilde{\vec{q}}_\perp \tilde{\vec{r}}_\perp - \kappa_{zl}\left(\tilde{\vec{q}}_\perp\right)z\right]\right\}d\tilde{\vec{q}}_\perp, \end{aligned} \qquad (20)$$

where $k_{zl}\left(\tilde{\vec{q}}_\perp\right)$ and $\kappa_{zl}\left(\tilde{\vec{q}}_\perp\right)$ are obtained from Eq. (13) with $\vec{k}_\perp$ and $\vec{\kappa}_\perp$ replaced by $\vec{q}_\perp$.

The equations for A_{l1} and B_{l1} are as follows

$$\sum_{m'=1}^{2L} S_{mm'}\eta_{m'} = \gamma_m\left(\tilde{\vec{k}}_\perp, \tilde{\vec{q}}_\perp\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right), \qquad (21)$$

$S_{mm'}$ has the same form as for the field of zero-approximation. The relationships between $\eta_{m'}$ and A_{l1}, B_{l1} are the same as for $\eta_{m'}$ and A_{l0}, B_{l0}:

$$\begin{aligned} \gamma_m\left(\tilde{\vec{k}}_\perp, \tilde{\vec{q}}_\perp\right) &= \sum_{l=1}^{L}\left[Q_{\beta m}^{(1)}\left(-\kappa_{zl}\right)B_{\beta 0} -\right. \\ &\left.- Q_{\alpha m}^{(1)}\left(-\kappa_{zl}\right)A_{l0} - Q_{\alpha m}^{(1)}\left(\kappa_{zl}\right)A_{l0}\right] + \\ &+ \tilde{\vec{k}}_\perp \tilde{\vec{q}}_\perp \sum_{l=1}^{L}\left[Q_{\beta m}^{(2)}\left(-\kappa_z\right)B_{l0} -\right. \\ &\left.- Q_{\alpha m}^{(2)}\left(-\kappa_z\right)A_{l0} - Q_{\alpha m}^{(2)}\left(k_z\right)A_{l0}\right]; \\ \tilde{\varsigma}\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right) &= \int \varsigma\left(\tilde{\vec{r}}_\perp\right)\exp\left[i\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right)\tilde{\vec{r}}_\perp\right]d\tilde{\vec{r}}_\perp. \end{aligned} \qquad (22)$$

The solution of this set has the form

$$\eta_{m'} = \Psi_{m'}\left(\tilde{\vec{k}}_\perp, \tilde{\vec{q}}_\perp\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right), \qquad (23)$$

where

$$\Psi_{m'} = \frac{1}{D}\sum_{m=1}^{2L} D_{m'm}\gamma_m\left(\tilde{\vec{k}}_\perp, \tilde{\vec{q}}_\perp\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right)$$

is equal to $\Psi_{\alpha l}$ at $l = 1...L$, and to $\Psi_{\beta l}$ at $l = L+1...2L$.

Therefore, we have

$$\begin{aligned} A_{l1} &= \Psi_{\alpha l}\left(\tilde{\vec{k}}_\perp, \tilde{\vec{q}}_\perp\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right),\ l = 1...L; \\ B_{l1} &= \Psi_{\beta l}\left(\tilde{\vec{k}}_\perp, \tilde{\vec{q}}_\perp\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_\perp - \tilde{\vec{q}}_\perp\right),\ l = L+1...2L. \end{aligned} \qquad (24)$$

As a result, we obtain

$$
\begin{aligned}
&E_{\alpha 1}=\sum_{l=1}^{L}\int \Psi_{\alpha l}\left(\tilde{\vec{k}}_{\perp},\tilde{\vec{q}}_{\perp}\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_{\perp}-\tilde{\vec{q}}_{\perp}\right)\times\\
&\times\exp\left\{i\left[\tilde{\vec{q}}_{\perp}\tilde{\vec{r}}_{\perp}+k_{zl}\left(\tilde{\vec{q}}_{\perp}\right)z\right]\right\}d\tilde{\vec{q}}_{\perp};\\
&E_{\beta 1}=\sum_{l=1}^{L}\int \Psi_{\beta l}\left(\tilde{\vec{k}}_{\perp},\tilde{\vec{q}}_{\perp}\right)\tilde{\varsigma}\left(\tilde{\vec{k}}_{\perp}-\tilde{\vec{q}}_{\perp}\right)\times\\
&\times\exp\left\{i\left[\tilde{\vec{q}}_{\perp}\tilde{\vec{r}}_{\perp}-\kappa_{zl}\left(\tilde{\vec{q}}_{\perp}\right)z\right]\right\}d\tilde{\vec{q}}_{\perp}.
\end{aligned}
\tag{25}
$$

Eqs. (25) describe the fields scattered on statistically rough surface in both half-spaces. In the case of the far zone, the integrals in Eqs. (25) can be calculated by the saddle-point technique. As the interface of the surfaces α and β is a stochastic function of coordinates, the scattered field is stochastic. The average fields $\langle E_{\alpha 1}\rangle$ and $\langle E_{\beta 1}\rangle$ are equal to zero.

Now the mean intensity of the scattered and transformed fields $\left\langle \left|E_{\alpha 1}\right|^2\right\rangle$ and $\left\langle \left|E_{\beta 1}\right|^2\right\rangle$ in the far zone can be calculated by way similar to made in [1]. The results are

$$
\left\langle \left|E_{\alpha 1}\right|^2\right\rangle=\frac{4\pi^2 S}{z^2}\sum_{l=1}^{L}\frac{\left|\Psi_{\alpha 1}\left(\tilde{\vec{q}}'_{\perp l},\tilde{\vec{k}}_{\perp}\right)\right|^2}{\left|\mu_{\alpha}\left(\tilde{\vec{q}}'_{\perp l}\right)\right|}\times \tilde{K}\left(\tilde{\vec{k}}_{\perp}-\tilde{\vec{q}}'_{\perp l}\right); \tag{26}
$$

$$
\left\langle \left|E_{\beta 1}\right|^2\right\rangle=\frac{4\pi^2 S}{z^2}\sum_{l=1}^{L}\frac{\left|\Psi_{\beta 1}\left(\tilde{\vec{q}}''_{\perp l},\tilde{\vec{k}}_{\perp}\right)\right|^2}{\left|\mu_{\beta}\left(\tilde{\vec{q}}''_{\perp l}\right)\right|}\times \tilde{K}\left(\tilde{\vec{k}}_{\perp}-\tilde{\vec{q}}''_{\perp l}\right), \tag{27}
$$

where $\tilde{\vec{q}}'_{\perp l}$ and $\tilde{\vec{q}}''_{\perp l}$ are the roots of the equations

$$
\frac{\partial k_{zl}\left(\tilde{\vec{q}}'_{\perp l}\right)}{\partial\tilde{\vec{q}}'_{\perp l}}=-\frac{\tilde{\vec{r}}_{\perp}}{z};\quad \frac{\partial \kappa_{zl}\left(\tilde{\vec{q}}'_{\perp l}\right)}{\partial\tilde{\vec{q}}'_{\perp l}}=\frac{\tilde{\vec{r}}_{\perp}}{z};
$$

$\tilde{K}\left(\tilde{\vec{\xi}}\right)=\int K\left(\tilde{\vec{\rho}}\right)\exp\left[i\left(\tilde{\vec{\xi}}\tilde{\vec{\rho}}\right)\right]d\vec{\rho}$ is the Fourier transformation of the correlation function;

$$
K\left(\tilde{\vec{\rho}}\right)=\left\langle \varsigma\left(\tilde{\vec{r}}_{\perp}\right)\varsigma\left(\tilde{\vec{r}}_{\perp}+\tilde{\vec{\rho}}\right)\right\rangle;
$$

$$
\mu_{\alpha}\left(\tilde{\vec{q}}'_{\perp}\right)=\frac{\partial^2 k_{zl}\left(\tilde{\vec{q}}_{\perp}\right)}{\partial q_x^2}\frac{\partial^2 k_{zl}\left(\tilde{\vec{q}}_{\perp}\right)}{\partial q_y^2}-\left(\frac{\partial^2 k_{zl}\left(\tilde{\vec{q}}'_{\perp}\right)}{\partial q_x\partial q_y}\right)^2;
$$

$$
\mu_{\beta}\left(\tilde{\vec{q}}''_{\perp}\right)=\frac{\partial^2 \kappa_{zl}\left(\tilde{\vec{q}}''_{\perp}\right)}{\partial q_x^2}\frac{\partial^2 \kappa_{zl}\left(\tilde{\vec{q}}''_{\perp}\right)}{\partial q_y^2}-\left(\frac{\partial^2 \kappa_{zl}\left(\tilde{\vec{q}}''_{\perp}\right)}{\partial q_x\partial q_y}\right)^2.
$$

The following conclusion about results obtained in this article can be made. The generalized method of small perturbations is applied to solve the problem of wave scattering and transformation from statistically rough surface separating media with a spatial and time dispersion. The problem cannot be solved by the usual small perturbation method which can be used only for non dispersive media, described by the Maxwell equations or by the wave equation [1, 2]. Since the media with dispersion are described by much more complicated equations and boundary conditions, the generalized method has been proposed.

1. *Bass F. G., Fuks I. M.* Wave scattering from statistically rough surface. – Oxford: Pergamon Press, 1979. – 537 p.
2. *Ogilvy J. A.* Theory of waves Scattering from random surface. – Bristol: Adam Hilger, 1991. – 277 p.
3. *Bass F. G., Freilikher V.* Surface scattering in media with time and spatial dispersion // Waves in Random and Complex media. – 2007. – 17, No. 2. – P. 213–220.
4. *Bass F. G., Freilikher V., Maradudin A. A.* Geometric optics of dispersive media with turning points // Waves in Random and Complex media. – 2008. – 18, No. 3. – P. 541–549.
5. *Bass F. G., Vatova L. B.* Wave diffraction on infinite chaotic screen bounding a medium with spatial and time dispersion // Waves in Random and Complex media. – 2007. – 17, No. 3. – P. 397–403.

ТРАНСФОРМАЦИЯ ВОЛН НА СТАТИСТИЧЕСКИ ШЕРОХОВАТОЙ ПОВЕРХНОСТИ

Ф. Г. Басс, Л. Б. Ватова

Обобщенный метод малых возмущений использован для исследования трансформации волн на статистически шероховатой поверхности, разделяющей среды, в которых из-за наличия временной и пространственной дисперсий распространяются несколько типов волн. Сформулированы для поля граничные условия. Рассчитана средняя интенсивность трансформированных волн.

Ключевые слова: статистически шероховатая поверхность, временная и пространственная дисперсии, трансформация волн, метод малых возмущений, метод перевала.

ТРАНСФОРМАЦІЯ ХВИЛЬ НА СТАТИСТИЧНО ШОРСТКІЙ ПОВЕРХНІ

Ф. Г. Басс, Л. Б. Ватова

Узагальнений метод малих збурень використано для дослідження трансформації хвиль на статистично шорсткій поверхні, що розділяє середовища, в яких із-за наявності часової та просторової дисперсій розповсюджуються декілька типів хвиль. Сформульовано для поля граничні умови. Розраховано середню інтенсивність трансформованих хвиль.

Ключові слова: статистично шорстка поверхня, часова та просторова дисперсії, трансформація хвиль, метод малих збурень, метод перевалу.

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