UDC 551.465

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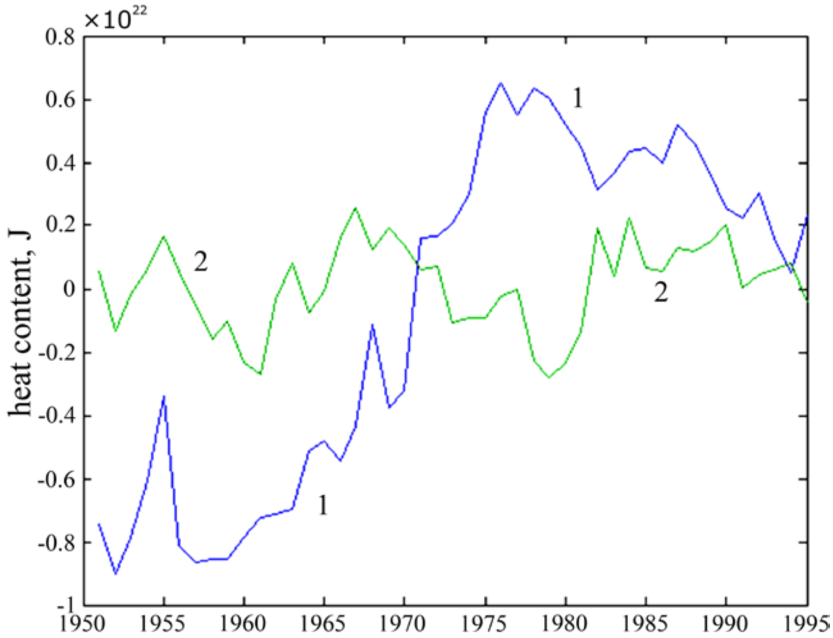
Non-linear pumping effect in oscillatory diffusive processes and its physical consequences for the World Ocean deep layers and lakes

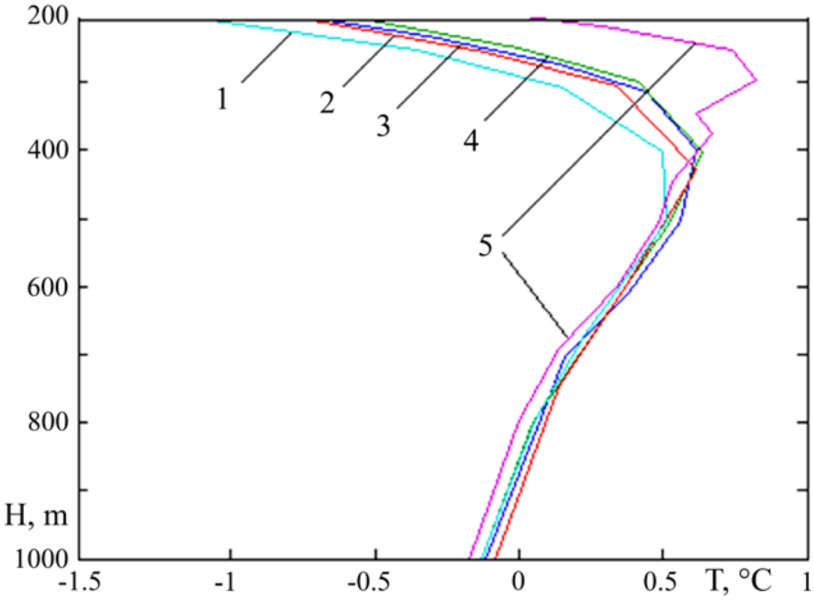
The physical mechanism of heat transmission in oscillatory processes is described. The effect manifestation in the process of integral heat exchange in the World Ocean deep layers is studied within the frame of a simple one-dimensional approach. The sea surface temperature (SST) has long-term oscillations with amplitudes greater than the trend in mean-temperature increase. The oscillations of SST lead to nonlinear pumping effect in oscillatory processes; heat is pumped out from or into the deep layers, depending on oscillation amplitudes. With increasing SST oscillation amplitudes, the heat comes out and deep layers are cooled, otherwise, with decreasing amplitudes, the heat spreads into the deep layers.

Keywords: pumping effect, heat transmission in oscillatory processes, sea surface temperature, one-dimensional model, deep layers of the World Ocean.

1. Introduction. At the moment, there is no clear explanation for the deep waters of the North Atlantic and Arctic basin getting colder. As shown in [1, 2], deep waters of the North Atlantic are cooler than before. The temperature at 1750 m depth in the North Atlantic decreased by –0.1 to –0.4°C in comparison with the period 1970 – 1974. Fig. 1 demonstrates heat content variations in ocean waters in the North Atlantic and North Pacific in the layer of 1000 – 3000 m. One can conclude that the deep waters became cooler in comparison with the period 1975 – 1980. The deep-water cooling is more pronounced in the Arctic. The interannual variability of the deep-water heat content near the Atlantic sector of the Arctic basin is 80% due to water layer change, whereas the long-term variability is caused by the water temperature change, accounting for 60% [3]. As reported in [4], the temperature of the Arctic basin deep waters decreased by –0.03°C from 1950 to 1978 and the maximum of the long-term water temperature decrease is –0.08°C at 1000 m in the period of observations up to 1998; at the same time, the temperature of upper layers (< 400 m) increased (Fig. 2). An interesting phenomenon is associated with the coldest waters of the World Ocean. In 1898, F. Nansen discovered waters with temperature of –1.3°C in the northern Norwegian Sea. Later, up to 1950, the temperature of these waters was not less than –1.1°C. However, starting from 1970, water temperature was quickly decreasing and in 1977 achieved a value of –1.2°C. In that period, the heat content of a 2 km deep layer decreased by more than 40 kcal/cm^2. At the same time, the average temperature of the bottom waters in the Arctic American – Asian sub-basin also decreased. During nearly 20 years, bottom water temperature in the Norwegian Sea decreased by –0.3°C, the maximal temperature of the deep Atlantic waters, by –0.1°C, and the average temperature of deep waters in the American – Asian sub-basin, by –0.05°C. The decrease in the temperature of the Barents branch of the Atlantic waters in the Arctic, a natural phenomenon still to be explained, is especially pronounced at the place where this branch merges with Fram's flow in the northern part of the Kara Sea. The Barents Current core became cooler by –0.2° to –0.4°C [5].



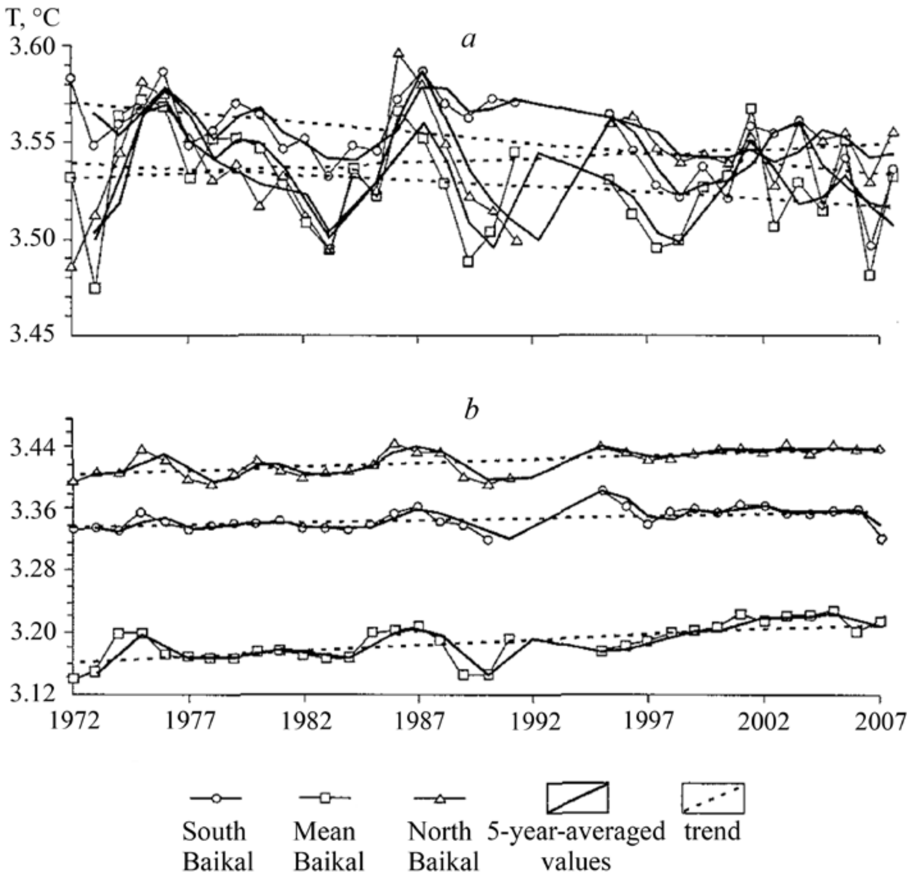
F i g. 1. Heat content variations of the deep water layer 1000 – 3000 m for years 1950 – 1995 in the North Atlantic (*1*) and the North Pacific (*2*) calculated by author from data of the paper [1]



F i g. 2. Distributions of temperature in the layer 200 – 1000 m in the Arctic Ocean [4]: *1 – 4* — in 1973 – 1976 years, *5* – in 1998 (the temperature at 1000 m decreased by about –0.08°C in 1998 in comparison with 1973 – 1976, but the temperature of upper layers (< 400 m) increased)

In the article [6] data of temperature and salinity observations in 1995 – 2004 on the parallel 24.5°N in the North Atlantic are described. The temperature of deep layers below 2000 m is shown to be decreased. A widespread idea is that cold penetrates into deep layers through downwelling of cooled surface waters in polar regions or in zones of deep convection as, for example, in the Irminger Sea [7]. If we proceed from this supposition, the deep layers of the World Ocean have to be warmer because in polar regions, the surface waters become warmer. As one can see from Fig. 2, the surface layers become warmer, but deep layers become cooler. So, it is impossible to explain the cooling of deep layers only by water convection in polar regions. Moreover, at the same time, the salinity of deep waters in the North Atlantic was found to increase. This does not agree with the idea of deep layers cooling by convection in polar regions, because ice melting leads to a decrease in surface salinity in the Arctic.

A more interesting situation is observed in Lake Baikal [8]. The surface water temperature in 1972 – 2007 was decreasing, while the temperature of deep layers in the lake was increasing (Fig. 3). Obviously, this phenomenon cannot be explained by convection.



F i g. 3. Averaged long-time temperature change in water layers in the South, Middle and North Baikal in June – September, 1972 – 2007 [8]: *a* – surface layer (200 – 400 m), *b* – bottom layer (200 m from bed)

From this, one can conclude that some other process can influence the temperature of deep layers in seas and lakes. In this article, it is shown that vertical turbulent diffusion of heat and salinity in oscillatory processes, the so-called pumping effect, can significantly influence the temperature of deep layers in seas and lakes.

2. Pumping effect for nonlinear parabolic equations. Before considering oceans and lakes, we expound briefly the theory of pumping effect. First this effect was described in [9]. Authors of the paper [10] did not know about this paper and obtained this effect independently, giving it the name of pumping effect.

Consider one-dimensional nonlinear thermal conductivity equation

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[F(T)\frac{\partial T}{\partial z}\right], \tag{1}$$

where $F(T)$ is the thermal conductivity «coefficient» (function), t – time, z is the spatial coordinate.

We search for a periodical solution of the equation (1) on a half-line $z > 0$ with the following boundary conditions:

$$T\big|_{z=0} = f(t) = T_0 + T_1 \cos\omega t, \quad T\big|_{z\to+\infty} < \overline{C} < \infty\ , \tag{2}$$

here $f(t)$ is a periodical function with period $\tau = 2\pi/\omega$, where ω is a frequency of oscillations.

The equation (1) with boundary conditions (2) describes many physical processes, e.g. the propagation of long waves on shallows, fluctuations of currents in porous mediums, propagation of temperature waves, polytropic gas, etc. [10].

Introduce an operator of averaging over period τ:

$$\langle T\rangle = \frac{1}{\tau}\int_t^{t+\tau} T\,dt \tag{3}$$

and the function $\Psi(T)$ as the primitive function of $F(T)$:

$$\Psi(T) = \int F(T)dT\ . \tag{4}$$

Assume $\Psi(T)$ is a single-valued function. Denote the inverse function to Ψ as $\Psi^{(-1)}$. Then, the following theorem is true [10]: periodical solution of equation (1) with boundary conditions (2) tends at $z \to +\infty$ to a constant $T^{(\infty)}$:

$$T^{(\infty)} = \Psi^{(-1)}\left[\left\langle \Psi(f(t))\right\rangle\right]. \tag{5}$$

Note that, in the general case, $T^{(\infty)}$ does not coincide with T_0. From (5) one can see that $\langle\Psi\rangle$ is invariant along z (the proof was adduced in Appendix 1).

Thus, pure harmonic oscillation of parameter T at the domain's boundary leads to an increase or decrease in T within the domain interior relative to the mean value at the boundary. Hence we observe an effect of either «pumping in» or «pumping out» of the substance at infinity caused by harmonic oscillation at the boundary.

It is easy to find the value of invariant $\langle \Psi \rangle$ at infinity, because oscillations attenuate there and equation (A1.4) can be used. However, in practice, such problem occurs frequently for limited regions and, when the problem is formulated over a limited segment $0 \le z \le L$, the procedure of finding the invariant considered in the previous section cannot be repeated at $z = L$. In the general case, equation (1) in a segment can be solved only numerically. However, if the ratio $\varepsilon = T_1 / T_0$ in the relation for $f(t)$ is small, i.e. $\varepsilon << 1$, it is possible to find an analytical expression for the pumping effect at the other end of the segment at $z = L$ (see Appendix 2).

Equations (A2.5), (A2.6) allow estimating the distance $L_{(+)}$, where the mean temperature approaches the asymptotic solution (A2.8) (see Appendix 2):

$$L_{(+)} = \frac{1}{\lambda + \lambda^*} = [2\,\mathrm{Real}(\lambda)]^{-1} = \left(\frac{\alpha}{2\omega}\right)^{1/2} = \left[\frac{F(T_0)}{2\omega}\right]^{1/2}. \quad (6)$$

If medium function $F(T)$ in equation (1) is a linear function $F(T) = \alpha + \beta T$ (as, for example, in the case of propagation of temperature waves in water, ice, or soil), we have the following relation for the pumping effect at the infinity:

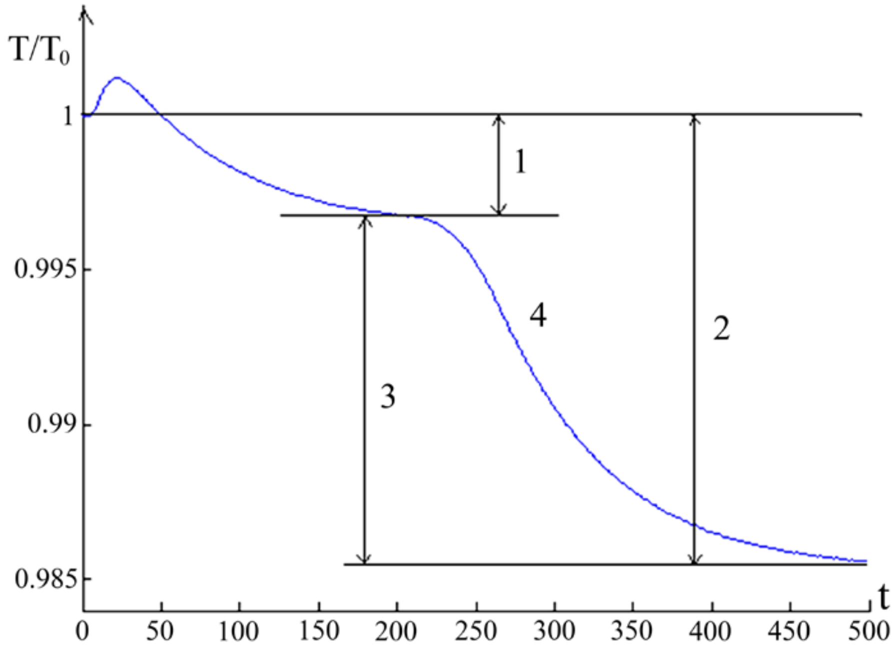
$$T^{(\pm)} = -b \pm \sqrt{b^2 + T_1^2 / 2}, \quad \text{where } b = \frac{\alpha}{\beta} + T_0. \quad (7)$$

If $b < 0$, one should take the minus sign in equation (7); if $b > 0$, the plus sign is taken; and if $T_1 / b << 1$ and $\alpha / \beta >> T_0$, equation (7) is simplified and reduced to equation (A2.8).

2.1. Numerical confirmation of the pumping effect. Consider the numerical model experiment to demonstrate the manifestation of pumping effect. Take function $F(T)$ in equation (1) in dimensionless form

$$F(T) = c\,a / (b + r\,T), \quad (8)$$

where parameters are equal $c = 10$, $a = 2.25$, $b = 1$, $r = 0.596$. Note that the presentation of function $F(T)$ in the form (8) is associated with the application of pumping effect to the ocean discussed in section 3 below. In the numerical calculations at the surface (z = 0), a periodic boundary condition $T = 1 + q(t)\sin(2\pi t / 5)$ is applied, t – is nondimensional parameter, the heat flux at the domain bottom (z = 5) is zero. We define the function $q(t)$ as follows: it is 0.2 at $0 < t < 200$ (the first regime); it linearly changes from 0.2 to 0.4 at $200 < t < 250$; and it is 0.4 at $250 < t < 500$ (the second regime). Using the function $q(t)$, we simulate a situation when the temperature at the ocean surface fluctuates with fixed amplitude up to a predefined time cutoff. After that, the fluctuation amplitude increases by a factor of two. As a result, the temperature in the lower part of the ocean (curve *4* in Fig. 4) reaches an asymptotic level corresponding to the first regime (level *1*), and later, when the fluctuation amplitude increases, the temperature reaches another asymptotic level, corresponding to the second regime, denoted as level *2*. The heat loss is determined by the difference between levels *1* and *2* (interval *3*).



F i g. 4. Behaviour of the temperature T/T_0 in time *t* (nondimensional) at the bottom of a model basin in the numerical experiment (see the text for explanations of *1 – 4* in section 2.1)

One may easily reach a general conclusion from physical considerations that the pumping effect is positive when $F(T)$ in (1) is an increasing function and negative when it is a decreasing function as, for example, the function (8). Indeed, the value in square brackets in equation (1) is heat flux. When the temperature at the boundary periodically changes and the function $F(T)$ increases during the phase of greater temperatures, the heat flux into the domain is greater than the flux out of it during the phase of temperature decrease. As a result, the net heat flux over the period is directed into the domain, leading to a positive pumping effect. Similar physical considerations for a decreasing function $F(T)$ lead to a negative effect of the heat exchange.

3. One-dimensional model for the World Ocean. The temperature T and the salinity S of the World Ocean averaged over the latitude and longitude were described by the following system of one-dimensional equations of nonlinear heat conductivity and salinity diffusion in the vertical direction:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K_T \frac{\partial T}{\partial z}\right),$$

$$\frac{\partial S}{\partial t} = \frac{\partial}{\partial z}\left(K_S \frac{\partial S}{\partial z}\right), \qquad (9)$$

where K_T and K_S are coefficients of thermal conductivity and salinity diffusion, respectively, z is the vertical coordinate. At the ocean surface the temperature and salinity oscillate periodically near their mean values

$$T\big|_{z=0} = T_0 + T_1 \cos(\omega t), \quad T\big|_{z\to+\infty} < C_1 < \infty\,,$$
$$S\big|_{z=0} = S_0 + S_1 \cos(\omega t), \quad S\big|_{z\to+\infty} < C_2 < \infty\,. \tag{10}$$

As known, ocean thermal conductivity and salinity diffusion are governed by the processes of turbulent mixing. Since processes of thermal conductivity and salinity diffusion were realized by turbulence, then it is reasonable to assume $K_T \sim K_S \sim K_U$, where K_U is the coefficient of vertical momentum exchange. Below the surface of the Ekman layer, the parameterization of K_U, generally accepted in oceanology [11 – 13], can be written as a function of the Brünt – Váisálá frequency $N(z) = \sqrt{\dfrac{g}{\rho_0}\dfrac{\partial\rho}{\partial z}}$ as

$$K_U(z) = \mu N^{-\gamma}, \tag{11}$$

where $\mu = \delta / N_0^{1-\gamma}$; $\delta \approx (1...2)\times 10^{-3}\ \mathrm{cm^2/s^2}$; $0.5 \le \gamma \le 1.5$; g – the acceleration of gravity, ρ – water density, ρ_0 is a mean value of ρ, and N_0 is the characteristic value of the Brünt – Váisálá frequency. As reported in [11], the most acceptable value is $\gamma = 1$. With weak stratification of the ocean deep layer, the value of $N(z)$ can be very small; as a result, the relation (11) overestimates the values of K_U. To avoid that, in [12] the modification of relation (31) was suggested by introducing an upper limit $K_U \le K_U^{\max}$. Then the equation (11) reads:

$$K_U(z) = \min\left(\mu N^{-\gamma}, K_U^{\max}\right). \tag{12}$$

With the assumption $K_U \le K_U^{\max}$, the relation (12) is valid over the entire oceanic water column. To apply directly the theory of pumping effect given in section 2, we have to express K_U as a function of not density gradient, but the density itself, i.e. in the form $K_U = K(\rho)$. For this purpose we use a hyperbolic law for the Brünt – Váisálá frequency below the Ekman layer in a geostrophic domain, obtained in [14],

$$N(z) = \frac{h_E N_E}{z + h_E}, \tag{13}$$

where h_E is the upper Ekman layer thickness and N_E is the value of Brünt – Váisálá frequency at the lower boundary of the Ekman layer. Note that the exponential Emery – Lee – Magaard approximation [15] or the exponential Munk – Wunsch approximation for a coefficient of a thermal conductivity [16] could be used instead of the hyperbolic approximation of Brünt – Váisálá frequency (13). However, comparison has shown the parameterization (13) to be in better agreement with the real distribution of Brünt – Váisálá frequency in the ocean. Moreover, the relationship (13) is preferable from the mathematical viewpoint.

So, taking into account (13), we can write

$$N^2(z) = \frac{g}{\rho_0}\frac{\partial \rho}{\partial z} = \left(\frac{h_E N_E}{z + h_E}\right)^2 . \tag{14}$$

A standard approximation for the Brünt – Váisálá frequency was made in (14) by substituting the mean value of ρ_0 instead of the density ρ in the denominator. Integrating (14) from the bottom $z = H$ to the level z, we find

$$\frac{h_E^2 N_E^2}{z + h_E} = \frac{g}{\rho_0}[\rho(H) - \rho(z)] + \frac{h_E^2 N_E^2}{H + h_E}, \tag{15}$$

where H is the ocean bottom. Substituting $h_E N_E/(z + h_E)$ from (15) in (13) and then in (11), we obtain

$$K_U(\rho) = \frac{\mu h_E^\gamma N_E^\gamma}{\left\{\frac{g}{\rho_0}\left[-\rho(z) + \rho(H)\right] + \frac{h_E^2 N_E^2}{H + h_E}\right\}^\gamma} . \tag{16}$$

We take the equation of the seawater state in the simple linear approximation as $\rho = \rho_H\left[1 - \alpha_T\left(T - T_H\right) + \beta_S(S - S_H)\right]$, where α_T is a coefficient of thermal expansion of water, β_S is a coefficient of salinity compression, $\rho_H = \rho(H)$, $T_H = T(H)$, $S_H = S(H)$. Multiplying the first equation of system (9) by $-\alpha_T$, and the second equation by β_S and adding them, we obtain the equation for the density $\rho(t, z)$:

$$\frac{\partial \rho}{\partial t} = \mathrm{Sc}\frac{\partial}{\partial z}\left[K(\rho)\frac{\partial \rho}{\partial z}\right], \tag{17}$$

where

$$K(\rho) = \frac{A}{(B - R\rho)^\gamma}\,;\; A = \mu h_E^\gamma N_E^\gamma;\; B = \frac{h_E^2 N_E^2}{H + h_E} - \frac{g\,\rho(H)}{\rho_0};\; R = \frac{g}{\rho_0}\,, \tag{18}$$

Sc is the Schmidt number, i.e. the ratio of the characteristic values of the coefficient of turbulent thermal conductivity to the kinematic coefficient of turbulent momentum exchange.

Substituting (10) into the equation of seawater state, we obtain boundary conditions for water density at ocean surface

$$\rho\big|_{z=0} = \rho_0 + \rho_1 \cos(\omega t), \quad \rho\big|_{z\to+\infty} < C_3 < \infty, \tag{19}$$

where

$$\rho_0 = \rho_H\left[1 - \alpha_T(T_0 - T_H) + \beta_S(S - S_H)\right],\; \rho_1 = \rho_H(\beta_S S_1 - \alpha_T T_1) . \tag{20}$$

In the case of a lake, $\beta_S = 0$ and we have the equation for the thermal conductivity

$$\frac{\partial T}{\partial t} = \mathrm{Sc}\frac{\partial}{\partial z}\left[\tilde{F}(T)\frac{\partial T}{\partial z}\right], \tag{21}$$

where

$$\tilde{F}(T) = \frac{\tilde{A}}{(\tilde{B}+\tilde{R}T)^{\gamma}};\ \tilde{A} = \mu h_E^{\gamma} N_E^{\gamma};\ \tilde{B} = \frac{h_E^2 N_E^2}{H+h_E} - \frac{g\,\rho_H\,\alpha_T}{\rho_0}T_H;\ \tilde{R} = \frac{g\,\rho_H\,\alpha_T}{\rho_0}. \tag{22}$$

3.1. Homogeneous ocean. First, consider a case of a homogeneous ocean $\rho(z) = \rho_0$. But the water density at the surface varies periodically $\rho = \rho_0 + \rho_1 \cos\omega t$ near a value ρ_0. The lower boundary of the surface Ekman layer (i.e. beginning of the geostrophic domain) is assumed to coincide with the ocean surface. The relation (12) allows us to avoid singularity in (17) for the real ocean, so we can consider $B - R\rho > 0$. An antiderivative function for $K(\rho)$ in (17) for $\gamma \neq 1$ is the function $-\dfrac{A}{(1-\gamma)R(B-R\rho)^{\gamma-1}}$, and for $\gamma = 1$ the function $(-A/R)\ln(B-R\rho)$. At $\gamma = 1$ we get the following expression for the value of the pumping effect:

$$\rho_\infty^{(+)} \approx \frac{R\rho_1^2}{4(B-R\rho_0)}. \tag{23}$$

Thus, one can see from (23) that if $\gamma = 1$ the pumping effect is positive. Calculations for $\gamma = 1/2$ and $\gamma = 2$ give the positive pumping effect for the density too. So, increasing amplitude of surface density oscillations leads to an increase in water density in the World Ocean deep layers.

If we assume $S(z) = \mathrm{const}$ then for the temperature we obtain the equation (21) with decreasing function $F(T)$. Therefore the pumping effect for temperature is negative. Similarly, one can find that the pumping effect for salinity is positive. So, the positive pumping effect for density corresponds to negative pumping effect for temperature and positive pumping effect for salinity.

Further we will consider the pumping effect for temperature alone. For the thermally homogeneous ocean $T(z) = T_0$ with periodic variability of water temperature at the surface $T = T_0 + T_1\cos\omega t$ with $\gamma = 1$, we get the following expression for the value of pumping effect

$$T_\infty^{(-)} \approx -\frac{RT_1^2}{4(B+RT_0)}. \tag{24}$$

So, for $\gamma = 1$ the pumping effect is negative for temperature and its value is two times smaller than for $\gamma = 2$. Finally, we can find an expression for the pumping effect for the other extreme case with $\gamma = 1/2$ (Appendix 3). Thus, for all values γ the pumping effect for temperature in the World Ocean is negative.

The layer, where the temperature oscillations are observed and after it the temperature almost reaches the pumping value $T^{(\pm)}$, is

$$L_{(+)} = \left[\frac{A}{2\omega(B+RT_0)^{\gamma}} \right]^{1/2} . \qquad (25)$$

This is the Stokes layer thickness. Assuming the Schmidt number $\mathrm{Sc} \approx 1$, we get $A \approx 2\times10^{-7}\,\mathrm{m}^3/\mathrm{s}^3$. From (25) one can obtain that Stokes layer thickness is about 83 m for oscillations with a period of one year and about 187 m for a period of five years. These estimates show that the thickness of Stokes layer, below which the temperature reaches the asymptotic level, is much less than the total thickness of the ocean; therefore, the approximation of an infinitely deep ocean is quite applicable to evaluating the pumping effect.

3.2. Effect of temperature decrease (increase) in the deep layers. As we see, the pumping effect in the World Ocean is negative for temperature for all admissible values of γ, while for density and salinity, it is positive. In addition, the value does not depend on A; hence, it does not depend on the Schmidt number Sc. Thus, when the amplitude of temperature fluctuations at surface T_1 increases, the temperature in the depths of the ocean decreases, that is, the heat is pumped out from the depths, and inversely, when the temperature fluctuations amplitude T_1 decreases as compared to the previous time period, the temperature in the ocean depths increases, i.e. heat spreads toward deeper layers. The result of the numerical model experiment shown in Fig. 4 illustrates this conclusion.

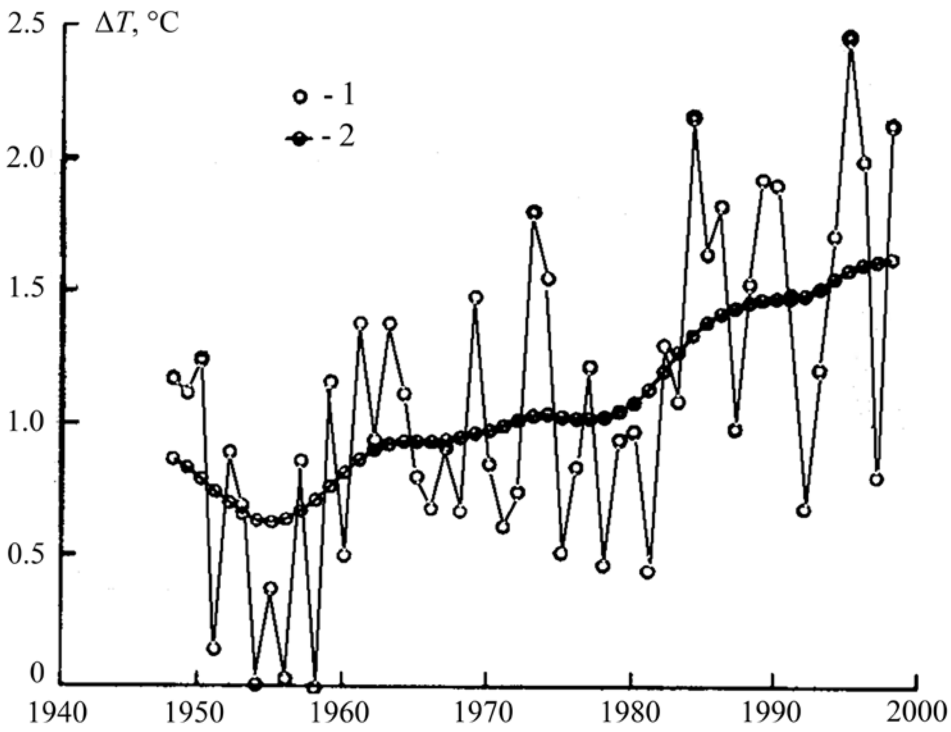
To estimate the amplitude increase of long-term fluctuations in the mean surface temperature of the World Ocean, we use the results of [17]. Following Reid, the relation for the globally averaged surface temperature of the World Ocean T_s and envelope curve of the solar activity fluctuations (Wolf number N_W) becomes

$$T_s = T_0 + \frac{0.089(1-\alpha)N_W}{4Q}, \qquad (26)$$

where $Q = 2.2\,\mathrm{W}/(\mathrm{m}^2\cdot{}^\circ\mathrm{C})$ and $\alpha = 0.3$ is the total albedo of the Earth. Analysis of fluctuations of the Solar activity (Wolf numbers) shows that, starting from 1925 – 1930, the solar activity amplitude fluctuation has been increasing. This is the upgoing phase of the 80 – 90 yrs Gleissberg period. The maximal value of the envelope curve of Wolf numbers during period 1900 – 1950 is 90 in average, while the same maximal value over 1950 – 2000 is about 190. Thus, the difference constitutes about 100. We suppose that the increase in long-term temperature fluctuations at the surface of the World Ocean is proportional to the increase in globally averaged surface temperature.

Substituting this value in (26), we get an estimate of the swing amplitude increase of long-term temperature fluctuations at the ocean surface $\Delta T_s \approx 0.7^\circ\,\mathrm{C}$. The amplitudes of the temperature fluctuations at the ocean surface for these periods are equal to half of the value assessed.

Another estimate of fluctuations of the mean surface temperature of the World Ocean can be derived from observational data. SST fluctuations in the equatorial zone of the Atlantic from 1950 is shown in Fig. 5 [18]. The graph is based on data of instrumental observations involving satellite imaginary. The amplitude increase of long-term SST fluctuations from 1960 is clearly seen in Fig. 5. By 2000, this increase reaches 1.5°C, i.e. it is two times greater than the estimate obtained from (26). So, an integral estimate of $\Delta T_s \approx 0.7^\circ\,\mathrm{C}$ seems reasonable. Moreover, Fig. 5 confirms the hypothesis that the long oscillation amplitude of the ocean surface temperature in 1950 – 2000 yrs increases by a factor of 1.5. We estimate the temperature decrease in the deep layers in case of a homogeneous ocean when the amplitude of long-term temperature fluctuations at surface exceeds $T_1 \approx 0.35^\circ\,\mathrm{C}$.



F i g. 5. Long-term temperature fluctuations *T* at the ocean surface in the equatorial zone of the Atlantic at the point (10°S; 0°) [18] (the value of $\Delta T = T - T_{\min}$ is plotted along the vertical axis, where $T_{\min}$ is the minimal temperature at the given point): *1* – data of observations; *2* – nonlinear trend from the paper [18]

Let us evaluate some applied parameter values. In the World Ocean, $h_E \approx 100\,\mathrm{m}, N_E \approx 10^{-2}\,\mathrm{s}^{-1}, H \approx 5\,\mathrm{km}$, and $\alpha_T = 1.67 \times 10^{-4} (^\circ\mathrm{C})^{-1}$. We assume the bottom temperature for the Pacific to be $T_H \approx 2.08^\circ\,\mathrm{C}$. Then, we get $\tilde{B} \approx -3.27 \times 10^{-3}\,\mathrm{m/s^2}$ and $\tilde{R} \approx 1.67 \times 10^{-3}\,\mathrm{m} / (\mathrm{s}^2 \cdot {}^\circ\mathrm{C})$. If we also assume that the mean deep water temperature in the Pacific, $T_0 \approx 3.66\,^\circ\mathrm{C}$ [19], is the mean-deep water temperature in the World Ocean, we get: $\tilde{B} + \tilde{R}T_0 \approx 2.8\times10^{-3}\ \mathrm{m/s^2}$.

Using relation (A3.1) in case of $\gamma = 2$, we get the following estimate for the temperature decrease in the deep ocean layers: $\Delta T_\infty^{(-)} = -3.6\times10^{-2}\,(^\circ\mathrm{C})$. Now we calculate the heat losses of the deep ocean layers over the period of climate warming. The estimated volume of the Pacific is $707.1\times10^6\,\mathrm{km}^3$ the temperature decrease is 0.036° C. The water thermal capacity is $0.928\ \mathrm{cal/(g\cdot{}^\circ C)}$. These values correspond to the heat of $707.1\times10^{21}\times0.928\times0.036 = 2.4\times10^{22}$ cal transported from the depths of the Pacific.

Since the volume of the Pacific is approximately 50% of the World Ocean, our estimate was doubled. Thus, over the period of climate warming, the heat loss of the World Ocean deep layers owing to the pumping effect at $\gamma = 2$ is about 4.8×10^{22} cal or 2.0×10^{23} J. Taking into account the heat transportation from the ocean during a period of 50 – 70 years and that the Earth's surface area is $5\times10^{14}\,\mathrm{m}^2$, we obtain the specific heat flux of $\approx 0.18 - 0.25$ W/m^2. This value is more than two times greater than the geothermal heat flux from the Earth's interior (0.09 W/m^2).

If $\gamma = 1$ the relation (24) reads: $\Delta T_\infty^{(-)} = -1.9\times10^{-2}\,(^\circ\mathrm{C})$. This is two times less than in the case of $\gamma = 2$, thus, in this case, all estimates obtained above have to be halved. In case of $\gamma = 1/2$, calculations from the relation (A3.4) lead to $\Delta T_\infty^{(-)} = -9\times10^{-3}\,(^\circ\mathrm{C})$.

4. Thermally non-homogeneous ocean. Estimates of the heat loss given above were made for a thermally homogeneous ocean. Such state corresponds to one of stationary solutions of equation (1), where the thermal conductivity coefficient is in the form (22) and the vertical heat flux is zero. If we admit that the heat flux is not zero, we can obtain a steady-state solution. For the case of $\gamma = 1$ in (21), we obtain a time-independent solution of equation (21):

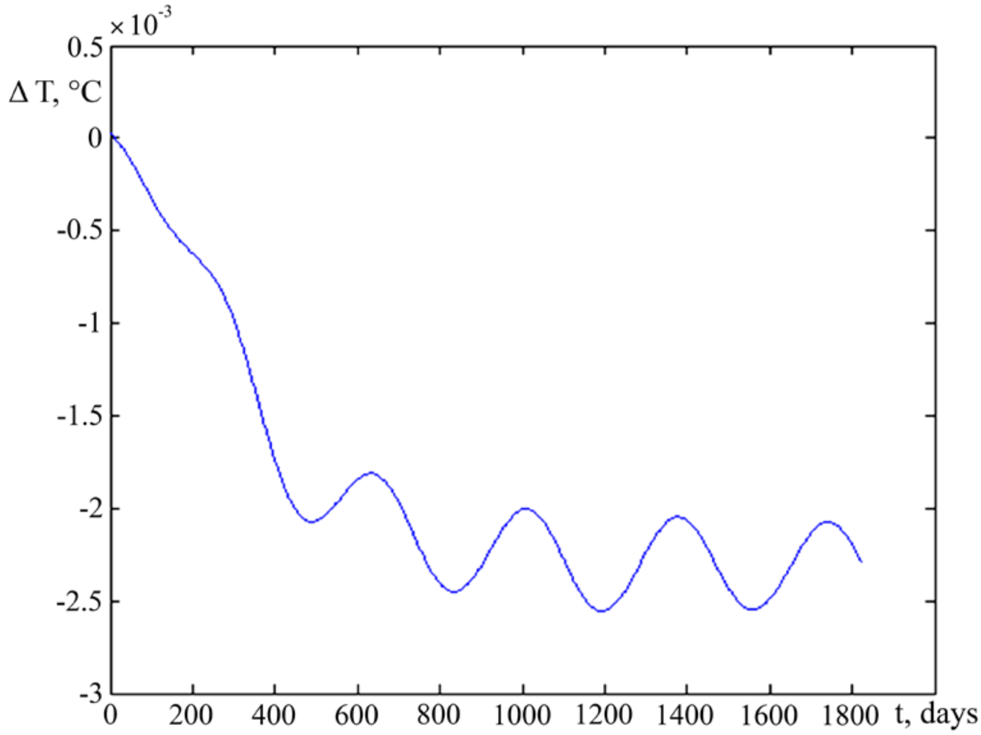
$$T(z) = \frac{1}{\tilde{R}}\left[-\tilde{B} + \tilde{C}_1 \exp(\tilde{C}_2 \tilde{R} z)\right], \tag{27}$$

where

$$\tilde{C}_1 = \tilde{R}T_s + \tilde{B}, \quad \tilde{C}_2 = \frac{1}{\tilde{R}H}\ln\frac{\tilde{R}T_H + \tilde{B}}{\tilde{R}T_s + \tilde{B}}, \tag{28}$$

T_s is the surface water temperature, T_H is its bottom temperature, and H is the depth of the fluid.

The solution (27), (28) describes a stationary exponential temperature distribution over ocean depth with surface temperature T_s and bottom temperature T_H. Fig. 6 gives a numerical solution of equation (1) with thermal conductivity function (22) at $\gamma = 1$. The stationary solution (27) with $T_s = 10^\circ$ C and $T_H = 2.08^\circ$ C is taken as the initial condition. On the ocean surface, the boundary condition $T(t,z)|_{z=0} = 10 + 0.7\sin\omega t$ with fluctuation period of $2\pi/\omega = 1$ year is used. The function of thermal conductivity has the form of (22) with $\tilde{B} = -3.27\times10^{-3}$ m/s^2, $\tilde{R} = 1.67\times10^{-3}$ m/(s^2·°C). Since coefficient A has no influence on the magnitude of the pumping effect and determines only the Stokes layer thickness, in these calculations we assumed $\tilde{A} = 4\times10^{-3}$ m^3/s^3. The amplitude of temperature fluctuations on the ocean surface was 0.7 according to Fig. 5. As one can see in Fig. 6, after the start of periodic variations in ocean surface temperature, the temperature in any point of water column goes down to low values. In accordance with this calculations, the temperature drop at the depth of 2000 m is 2.5×10^{-3} (°C).



F i g. 6. Temperature drop at the depth 2000 m relative to the stationary solution (equation (19)) after the start of the annual oscillation of ocean surface temperature (numerical solution)

Now let us consider the case of Lake Baikal. In Fig. 3 long-term changes of temperature of surface and bottom layers in Baikal are shown. From Fig. 3, *a* it can be seen, that the average temperature of surface layers of the lake over period 1972 – 2007 decreased, i.e. surface waters were cooled. At the same time, the temperature of bottom water layers rose (Fig. 3, *b*). It is impossible to explain the rise of bottom water temperature by the convection process. The only explanation that can be offered for this phenomenon is based on the pumping effect. Indeed, as can be seen from Fig. 3, *a*, the amplitude of long-term fluctuations of surface water temperature in period 1972 – 2007 almost halved. Under the pumping effect theory, a decrease in the amplitude of surface water temperature fluctuations in the lake will lead to pumping in of heat in deep layers, i.e. to an increase in the temperature of deep waters.

5. Conclusions. The estimates obtained in this study show that the pumping effect can serve as a strong nonlinear mechanism in processes of heat flux redistribution on the Earth. As follows from the results described above, a part of the heat that warmed the atmosphere during the periods of climate warming is transported from the deep layers of the ocean. According to different estimates, the magnitude of greenhouse effect is equal to the increase in additional heat flux to the Earth surface from 0.35 to 4 – 5 W/m^2. The additional heat flux from the deep layers of the World Ocean as a result of the pumping effect is estimated at about 0.18 – 0.25 W/m^2. It is comparable with the lower value of the flux caused by greenhouse effect. If we assume even higher values of SST long-term fluctuations [18], the estimates of the pumping effect magnitude become higher also.

Thus, the deep layers of the ocean serve as heat storage (accumulator) of the Earth when the amplitude of long-term temperature fluctuations at the ocean surface increases. This situation is observed during the past 50 years. Due to the negative pumping effect for temperature in the ocean, the heat is pumped up from the deep ocean to the atmosphere; on the contrary, during period of SST amplitude decrease, heat penetrates into the deep layers. A hypothesis explains the increase in SST fluctuations amplitude by the existence of solar activity envelope curve [20]. With this hypothesis, one can explain the appearance of 1950 – 1970 cooling period [21]. During this period, the solar activity decreased. Following to pumping effect theory, a decrease in SST fluctuations amplitude leads to pumping-down of heat from ocean surface to the deep ocean during the period of 1950 – 1970 and, consequently, to a slight climate cooling.

As a result, the nonlinear pumping effect is a cause of heat exchange between the ocean and the atmosphere. In climate-warming period, the amplitude of SST oscillations increases, causing cooling of the deep layers. This idea was discussed in paper [22]. Our estimates show that, for the Earth climate warming period, the deep layers of the World Ocean may become cooler by $(0.9 – 3.6) \times 10^{-2}$ (°C). Other manifestations of pumping effect in geophysics were considered in the work [23].

Lake Baikal demonstrates another manifestation of the pumping effect. During the period 1972 – 2007, the amplitude of long-time oscillations of surface temperature decreased and the temperature of deep layers in the lake increased as it should be according the pumping effect theory.

Acknowledgements This study was supported by the Russian Foundation for Basic Research (project no. 13-05-00131).

Appendix 1

Taking into account equation (4), we can rewrite equation (1) as

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left[\frac{d\Psi}{dT}\frac{\partial T}{\partial z}\right] \equiv \frac{\partial^2 \Psi}{\partial z^2}. \quad \text{(A1.1)}$$

Averaging the left- and right-hand parts of equation (A1.1) over period τ yields

$$\frac{d^2\langle\Psi\rangle}{dz^2} = 0 \quad \text{(A1.2)}$$

and, consequently, $\langle\Psi\rangle = C_1 z + C_2$. Since $\langle\Psi\rangle$ is nothing but heat flux averaged over the period, $\langle\Psi\rangle$ cannot grow infinitely at $z \to +\infty$, therefore $C_1 = 0$. It follows from this that $\langle\Psi\rangle = C_2$ and $\langle\Psi\rangle$ is an invariant independent of z. As a result, we get

$$\langle\Psi\rangle\big|_{z=0} = \langle\Psi\rangle_{z\to+\infty}. \quad \text{(A1.3)}$$

At $z \to +\infty$, oscillations attenuate and we have at infinity

$$\langle\Psi\rangle_{z\to+\infty} = \Psi(T^{(\infty)}). \quad \text{(A1.4)}$$

Taking into account that

$$\langle\Psi\rangle\big|_{z=0} = \langle\Psi(f(t))\rangle \quad \text{(A1.5)}$$

and using the inverse function $\Psi^{(-1)}$ to Ψ in (A1.4), we obtain relation (5) from (A1.3) and (A1.5).

Appendix 2

Consider equation (1) with boundary conditions (2) and an expansion of *F*(*T*) into a series to the terms of the first order with respect to ε:

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left\{[\alpha + \beta\,\varepsilon T + O(\varepsilon)]\frac{\partial T}{\partial z}\right\}, \quad \text{(A2.1)}$$

where $\alpha = F(T_0)$, $\beta = \dfrac{dF(T_0)}{dT}$. At the right end of the segment, we specify the boundary condition of the second kind

$$\left.\frac{\partial T}{\partial z}\right|_{z=L} = 0, \quad \text{(A2.2)}$$

which physically corresponds to the condition of zero thermal flux. We search for the solution of equation (A2.1) in the form of asymptotic expansion $T = T^{(0)} + \varepsilon T^{(1)} + ...$ with respect to ε with boundary conditions

$$T^{(0)}\Big|_{z=0} = A\cos\omega t\,, \quad \left.\frac{\partial T^{(0)}}{\partial z}\right|_{z=L} = 0\,, \quad T^{(1)}\Big|_{z=0} = 0\,, \quad \left.\frac{\partial T^{(1)}}{\partial z}\right|_{z=L} = 0\,. \qquad \text{(A2.3)}$$

We search for the solution for the first approximation $T^{(0)}$ as

$$T^{(0)} = \mathrm{Real}\left[Q(z)\mathrm{e}^{i\omega t}\right] = \frac{Q(z)\,\mathrm{e}^{i\omega t} + Q^*(z)\,\mathrm{e}^{-i\omega t}}{2}\,, \qquad \text{(A2.4)}$$

where Real denotes the real part and the asterisk denotes complex conjugated function. Substituting (A2.4) into the first approximation of equation (A2.1), we obtain the solution for $Q(z)$:

$$Q(z) = T_1\,\frac{\cosh[\lambda(L-z)]}{\cosh(\lambda L)}\,, \qquad \text{(A2.5)}$$

where $\lambda = (1+i)\sqrt{\omega/(2\alpha)}$. Substituting (A2.5) into (A2.4) and then into the second approximation of equation (A2.1) with respect to ε, we obtain a solution for $T^{(1)}$, containing a periodical part and a time-independent additive, which describes the pumping effect:

$$T^{(\pm)}(z) = -\frac{\beta}{4\alpha}\left[Q(z)Q^*(z) - Q(0)Q^*(0)\right]. \qquad \text{(A2.6)}$$

Equation (A2.6) gives a quantitative value of the pumping effect at point *z*. At the end of segment $z = L$, the pumping effect will be

$$T^{(\pm)}(L) = -\frac{\beta T_1^2}{4\alpha}\left[\frac{1}{\cosh(\lambda L)\cosh(\lambda^* L)} - 1\right]. \qquad \text{(A2.7)}$$

At $L \to \infty$, we get

$$T^{(\pm)}(\infty) = \frac{\beta T_1^2}{4\alpha}\,. \qquad \text{(A2.8)}$$

As can be seen from (A2.8), the sign of pumping effect depends on the sign of β/α.

Appendix 3

In the case of $\gamma = 2$ we get the expression for pumping effect

$$T_\infty = \frac{1}{\tilde{R}}\left[\sqrt{\left(\tilde{B} + \tilde{R}T_0\right)^2 - \left(\tilde{R}T_1\right)^2} - \tilde{B}\right]. \qquad \text{(A3.1)}$$

If we represent T_∞ as $T_\infty = T_0 + T_\infty^{(\pm)}$, then, using (A3.1), we can write with the condition $\tilde{R}T_1/\left(\tilde{B} + \tilde{R}T_0\right) << 1$:

$$T_\infty^{(-)} \approx -\frac{\tilde{R}T_1^2}{2\left(\tilde{B} + \tilde{R}T_0\right)}. \qquad \text{(A3.2)}$$

The case $\gamma = 1/2$ is the most labor-intensive for numerical calculations. In this case, an antiderivative function is the function $\frac{2\tilde{A}}{\tilde{R}}\sqrt{\tilde{B} + \tilde{R}T}$. We get the following expression:

$$T_\infty^{(\pm)} = \frac{1}{\tilde{R}}\left[\frac{16}{4\pi^2}\left(\tilde{B} + \tilde{R}T_0 + \tilde{R}T_1\right)E^2(k) - \left(\tilde{B} + \tilde{R}T_0\right)\right], \qquad \text{(A3.3)}$$

where $k^2 = 2a/(1+a) < 1$, $a = \tilde{R}T_1/\left(\tilde{B} + \tilde{R}T_0\right)$ and $E(k)$ is a full elliptic integral of the second type.

Raising the elliptic integral $E(k)$ to the second power and leaving only the terms up to the fourth order of magnitude respectively to k, we get:

$$T_\infty^{(-)} \approx -\frac{\tilde{R}T_1^2}{8\left(\tilde{B} + \tilde{R}T_0 + \tilde{R}T_1\right)}. \qquad \text{(A3.4)}$$

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Received July 11, 2012

АНОТАЦІЯ Описаний фізичний механізм передачі тепла в ході коливальних процесів. У рамках простої одновимірної моделі демонструється прояв цього ефекту в процесі інтегрального теплообміну в глибинних шарах Світового океану. Спостерігаються довготривалі коливання температури поверхні моря, амплітуда яких перевищує тренд при зростанні середньої температури. Коливання поверхневої температури унаслідок нелінійності призводять до ефекту накачування: тепло викачується з глибинних шарів або поступає в них залежно від амплітуд коливань. Із збільшенням амплітуди коливань температури поверхні моря тепло виходить з глибинних шарів і вони остуджуються, а при зменшенні амплітуди тепло розповсюджується в глибинні шари.

Ключові слова: ефект накачування, передача тепла в коливальних процесах, температура поверхні моря, одновимірна модель, глибинні шари Світового океану.

АННОТАЦИЯ Описан физический механизм передачи тепла в ходе колебательных процессов. В рамках простой одномерной модели демонстрируется проявление этого эффекта в процессе интегрального теплообмена в глубинных слоях Мирового океана. Наблюдаются долговременные колебания температуры поверхности моря, амплитуда которых превышает тренд при возрастании средней температуры. Колебания поверхностной температуры вследствие нелинейности приводят к эффекту накачки: тепло выкачивается из глубинных слоев или поступает в них в зависимости от амплитуд колебаний. С увеличением амплитуды колебаний температуры поверхности моря тепло выходит из глубинных слоев и они остужаются, а при уменьшении амплитуды тепло распространяется в глубинные слои.

Ключевые слова: эффект накачки, передача тепла в колебательных процессах, температура поверхности моря, одномерная модель, глубинные слои Мирового океана.